Contents

1. Measures of Possibility and Fuzzy Sets **1**

1.1. Imprecision and Uncertainty 1
1.2. Traditional Models of Imprecision and Uncertainty 4
1.3. Confidence Measures 7
1.3.1. Measures of Possibility and of Necessity 8
1.3.2. Possibility and Probability 10
1.4. Fuzzy Sets 13
1.5. Elementary Fuzzy Set Operations 17
1.6. Practical Methods for Determining Membership Functions . 19
1.6.1. Vague Categories as Perceived by an Individual . . 19
1.6.2. Fuzzy Sets Constructed from Statistical Data 20
1.6.3. Remarks on the Set of Degrees of Membership . . 24
1.7. Confidence Measures for a Fuzzy Event 25
1.8. Fuzzy Relations and Cartesian Products of Fuzzy Sets . . . 27
References 30

2. The Calculus of Fuzzy Quantities **33**

2.1. Definitions and a Fundamental Principle 33
2.1.1. Fuzzy Quantities, Fuzzy Intervals, Fuzzy Numbers . 33
2.1.2. The Extension Principle 36
2.2. Calculus of Fuzzy Quantities with Noninteractive Variables 38
2.2.1. Fundamental Result 39
2.2.2. Relation to Interval Analysis 42
2.2.3. Application to Standard Operations 43

2.2.4. The Problem of Equivalent Representations of a Function 47
2.3. Practical Calculation with Fuzzy Intervals 48
2.3.1. Parametric Representation of a Fuzzy Interval . . . 48
2.3.2. Exact Practical Calculation with the Four Arithmetic Operations 50
2.3.3. Approximate Calculation of Functions of Fuzzy Intervals 52
2.4. Further Calculi of Fuzzy Quantities 53
2.4.1. "Pessimistic" Calculus of Fuzzy Quantities with Interactive Variables 54
2.4.2. "Optimistic" Calculus of Fuzzy Quantities with Noninteractive Variables 56
2.5. Illustrative Examples 57
2.5.1. Estimation of Resources in a Budget 57
2.5.2. Calculation of a PERT Analysis with Fuzzy Duration Estimates 60
2.5.3. A Problem in the Control of a Machine Tool . . . 63
Appendix: Computer Programs 66
References 71

3. The Use of Fuzzy Sets for the Evaluation and Ranking of Objects 73

3.1. A Quantitative Approach to Multiaspect Choice 73
3.1.1. Basic Principles of the Approach 74
3.1.2. Fuzzy Set-Theoretic Operations 77
3.1.3. Application to the Combination of Criteria 86
3.1.4. Identification of Operators 94
3.1.5. Example 95
3.2. Comparison of Imprecise Evaluations 98
3.2.1. Comparison of a Real Number and a Fuzzy Interval 99
3.2.2. Comparison of Two Fuzzy Intervals 101
3.2.3. Ordering of n Fuzzy Intervals 105
3.2.4. Computer Implementation 106
3.2.5. Example 107
Appendix: Computer Programs 108
References 113

4. Models for Approximate Reasoning in Expert Systems 117

4.1. Remarks on Modeling Imprecision and Uncertainty 117
4.1.1. Credibility and Plausibility 118
4.1.2. Decomposable Measures 121

4.1.3. Vague Propositions 123
4.1.4. Evaluating the Truth Value of a Proposition 123
4.2. Reasoning from Uncertain Premises 127
4.2.1. Deductive Inference with Uncertain Premises . . . 130
4.2.2. Complex Premises 136
4.2.3. Combining Degrees of Uncertainty Relative to the Same Proposition 136
4.3. Inference from Vague or Fuzzy Premises 142
4.3.1. Representation of the Rule "if X is A, then Y is B" 142
4.3.2. "Generalized" Modus Ponens 143
4.3.3. Complex Premises 146
4.3.4. Combining Possibility Distributions 146
4.4. Brief Summary of Current Work and Systems 147
4.5. Example 148
Appendix A 152
Appendix B: Computer Programs 158
References 175

5. Heuristic Search in an Imprecise Environment, and Fuzzy Programming 181

5.1. Heuristic Search in an Imprecise Environment 182
5.1.1. A and A^* Algorithms 182
5.1.2. The Classical Traveling Salesman Problem (Reminder) 183
5.1.3. Heuristic Search with Imprecise Evaluations 185
5.1.4. Heuristic Search with Fuzzy Values 189
5.2. An Example of Fuzzy Programming: Tracing the Execution of an Itinerary Specified in Imprecise Terms 191
5.2.1. Execution and Chaining of Instructions 193
5.2.2. Illustrative Example 195
5.2.3. Problems Arising in Fuzzy Programming 202
5.2.4. Concluding Remarks 207
Appendix: Computer Programs 208
A.1. Selection of "the Smallest" of N Fuzzy Numbers . . 208
A.2. Tracing Imprecisely Specified Itineraries 209
References 214

6. Handling of Incomplete or Uncertain Data and Vague Queries in Database Applications 217

6.1. Representation of Incomplete or Uncertain Data 218
6.1.1. Representing Data by Means of Possibility Distributions 218

6.1.2. Differences and Similarities with Other Fuzzy Approaches 221
6.1.3. Dependencies and Possibilistic Information 224
6.2. The Extended Relational Algebra and the Corresponding Query Language 225
6.2.1. Generalization of Θ-Selection 225
6.2.2. Cartesian Product, Θ-Join, and Projection 232
6.2.3. Union and Intersection—Redundancy 235
6.2.4. Queries Employing Other Operations 236
6.3. Example 237
6.3.1. Representation of Data 237
6.3.2. Examples of Queries 240
6.4. Conclusion 243
Appendix: Computer Program 244
A.1. Data Structures 244
A.2. Representation of Queries 246
A.3. Description of Implemeted Procedures 247
References 257

Index **261**