

Contents

Preface	ix
1 Introduction	1
2 Basic Finite Element Concepts	7
2.1 Model Problem	7
2.2 Mesh-Based Elements	9
2.3 Sobolev Spaces	12
2.4 Abstract Variational Problems	16
2.5 Approximation Error	19
3 B-Splines	23
3.1 The Concept of Splines	23
3.2 Definition and Basic Properties	25
3.3 Recurrence Relation	27
3.4 Representation of Polynomials	29
3.5 Subdivision	32
3.6 Scalar Products	35
4 Finite Element Bases	37
4.1 Multivariate B-Splines	37
4.2 Splines on Bounded Domains	40
4.3 Weight Functions	43
4.4 Web-Splines	46
4.5 Hierarchical Bases	50
5 Approximation with Weighted Splines	53
5.1 Dual Functions	53
5.2 Stability	56
5.3 Polynomial Approximation	60
5.4 Quasi-Interpolation	63
5.5 Boundary Regularity	65
5.6 Error Estimates for Standard Weight Functions	67

6	Boundary Value Problems	71
6.1	Essential Boundary Conditions	71
6.2	Natural Boundary Conditions	75
6.3	Mixed Problems with Variable Coefficients	79
6.4	Biharmonic Equation	82
6.5	Linear Elasticity	86
6.6	Plane Strain and Plane Stress	89
7	Multigrid Methods	95
7.1	Multigrid Idea	95
7.2	Grid Transfer	99
7.3	Basic Algorithm	103
7.4	Smoothing and Coarse Grid Approximation	105
7.5	Convergence	108
8	Implementation	113
8.1	Boundary Representation	114
8.2	Classification of Grid Cells	116
8.3	Evaluation of Weight Functions	119
8.4	Numerical Integration	124
8.5	Matrix Assembly	127
	Appendix	131
	Notation and Symbols	135
	Bibliography	137
	Index	143