

Contents

Preface to the English Edition	XIII
Preface to the German Edition	XV
1 The Mathematical Framework	1
1.1 Hilbert Vector Space	2
1.1.1 The Scalar Product and the Dirac Notation	2
1.1.2 Linear Operators on the Hilbert Space	3
1.1.3 Normal Operators and Spectral Decompositions	6
1.1.4 Hermitian Operators	9
1.1.5 Unitary Operators	11
1.1.6 Positive Operators and Projection Operators	11
1.2 Liouville Operator Space	13
1.2.1 Scalar Product	13
1.2.2 Superoperators	14
1.3 The Elements of Probability Theory	15
1.3.1 The Probability of Random Events	15
1.3.2 Conditional Probability and Bayes' Theorem	17
1.3.3 Random Quantities	19
1.4 Complementary Topics and Further Reading	19
1.5 Problems for Chapter 1	20
2 Basic Concepts of Quantum Theory	23
2.1 First Version of the Postulates (pure states of isolated quantum systems)	23
2.1.1 Introduction: the Scenario of Quantum Mechanics	23
2.1.2 Quantum States	29
2.1.3 Postulates for Pure States of Isolated Quantum Systems	32
2.1.4 Comments on the Postulates	35
2.2 Outlook	37
2.3 Manipulation of the Evolution of the States by Projective Measurements	38
2.3.1 The Quantum Zeno Effect	38
2.3.2 Driving a State Vector by a Sequence of Projection Measurements	39
2.4 The Structure of Physical Theories*	40

The chapters marked with an asterisk * can be skipped over in a first reading.

2.4.1	Structural Elements of a Physical Theory*	41
2.4.2	Developed Reality*	42
2.5	Interpretations of Quantum Theory and Physical Reality*	43
2.5.1	The Minimal Interpretation*	43
2.5.2	The Standard Interpretation*	44
2.6	Complementary Topics and Further Reading	46
3	The Simplest Quantum Systems: Qubits	49
3.1	Pauli Operators	50
3.2	Visualisation of Qubits on the Bloch Sphere	52
3.3	Visualisation of the Measurement Dynamics and the Unitary Dynamics	55
3.4	Quantum Gates for Single Qubit Systems	59
3.5	Spin- $\frac{1}{2}$	62
3.6	Photon Polarisations	62
3.7	Single Photons in a Beam Splitter and in an Interferometer	63
3.7.1	Beam Splitters	64
3.7.2	Interferometer	66
3.8	Locating a Bomb Without Exploding It by Using a Null Measurement*	68
3.9	Complementary Topics and Further Reading	71
3.10	Problems for Chapter 3	71
4	Mixed States and the Density Operator	73
4.1	Density Operators for a Given Ensemble (Statistical Mixture)	73
4.1.1	Pure States	73
4.1.2	The Physics of Statistical Mixtures (Blends)	75
4.1.3	Definition and Properties of the Generalised Density Operator	78
4.1.4	Incoherent Superpositions of Pure States	80
4.2	The Generalised Quantum State	82
4.3	Different Ensemble Decompositions of a Density Operator and the Ignorance Interpretation	82
4.4	Density Operators of Qubits	85
4.5	Complementary Topics and Further Reading	86
4.6	Problems for Chapter 4	86
5	Shannon's Entropy and Classical Information	89
5.1	Definition and Properties	89
5.2	Shannon's Theorem	93
5.2.1	Typical Sequences	93
5.2.2	Classical Data Compression	95
5.3	Classical Information	96
5.4	Classical Relative Entropy	97
5.5	Mutual Information as a Measure of the Correlation between Two Messages	97
5.5.1	Mutual Information	98
5.5.2	Conditional Entropy	99
5.6	Complementary Topics and Further Reading	101
5.7	Problems for Chapter 5	101

6 The von Neumann Entropy and Quantum Information	103
6.1 The Quantum Channel and Quantum Entropy	103
6.2 Qubits as the Unit of Quantum Information	106
6.3 Properties	108
6.4 The Interfaces of Preparation and Measurement	110
6.4.1 The Entropy of Projective Measurements	110
6.4.2 The Entropy of Preparation	111
6.5 Quantum Information	112
6.6 Complementary Topics and Further Reading	112
6.7 Problems for Chapter 6	113
7 Composite Systems	115
7.1 Subsystems	115
7.2 The Product Hilbert Space	116
7.2.1 Vectors	116
7.2.2 Operators	118
7.3 The Fundamentals of the Physics of Composite Quantum Systems	120
7.3.1 Postulates for Composite Systems and Outlook	120
7.3.2 The State of a Subsystem, the Reduced Density Operator, and General Mixtures	122
7.4 Manipulations on a Subsystem	124
7.4.1 Relative States and Local Unitary Transformations	124
7.4.2 Selective Local Measurements	125
7.4.3 A Non-Selective Local Measurement	127
7.5 Separate Manipulations on both Subsystems	128
7.5.1 Pairs of Selective Measurements	128
7.5.2 Non-Local Effects: “Spooky Action at a Distance”?	130
7.6 The Unitary Dynamics of Composite Systems	131
7.7 A First Application of Entanglement: a Conjuring Trick	132
7.7.1 The Conjuring Trick	132
7.7.2 Classical Correlations can give no Explanation	133
7.7.3 The Trick	134
7.8 Quantum Gates for Multiple Qubit Systems	135
7.8.1 Entanglement via a CNOT Gate	135
7.8.2 Toffoli, SWAP, and Deutsch Gates	137
7.9 Systems of Identical Particles*	138
7.10 Complementary Topics and Further Reading	141
7.11 Problems for Chapter 7	142
8 Entanglement	143
8.1 Correlations and Entanglement	143
8.1.1 Classically-Correlated Quantum States and LOCC	143
8.1.2 Separability and Entanglement	145
8.1.3 The Separability Problem	147
8.2 Outlook	148

8.3	Entangled Pure States	149
8.3.1	The Schmidt Decomposition	149
8.3.2	The Schmidt Number and Entanglement	151
8.3.3	The Entropy of the Subsystems as a Measure of Entanglement	152
8.3.4	Subsystems in Pure States are not Entangled*	153
8.4	The PPT Criterion for the Entanglement of Mixtures *	155
8.5	The Production of Entangled States	157
8.6	The No-Cloning Theorem Prevents Transfer of Information Faster than the Velocity of Light	159
8.7	Marking States by Entanglement*	161
8.7.1	Which-Way Marking*	161
8.7.2	Quantum Erasure*	164
8.7.3	Delayed Choice of the Marker Observables*	165
8.8	Complementary Topics and Further Reading	166
8.9	Problems for Chapter 8	167
9	Correlations and Non-Local Measurements	169
9.1	Entropies and the Correlations of Composite Quantum Systems	169
9.1.1	Mutual Information as a Measure of Correlations	169
9.1.2	The Triangle Inequality	170
9.1.3	Entangled vs. Classically-Correlated Quantum Systems	171
9.2	Non-Local Measurements	174
9.2.1	The Bell Basis	174
9.2.2	Local and Non-Local Measurements	175
9.2.3	Non-local Measurements by Means of Local Measurements on an Ancillary System	177
9.2.4	Non-locally Stored Information and Bell Measurements	179
9.3	Complementary Topics and Further Reading	181
9.4	Problems for Chapter 9	181
10	There is no (Local-Realistic) Alternative to Quantum Theory	183
10.1	EPR Experiments and Their Quantum-mechanical Explanation	183
10.2	Correlated Gloves	186
10.3	Local Realism	187
10.4	Hidden Variables, Bell Inequalities and Contradictions of Experiments	188
10.5	Separable Mixtures Obey the Bell Inequality	191
10.6	Entanglement Witnesses*	192
10.7	3-Particle Entanglement and Quantum Locality	194
10.7.1	The GHZ State	194
10.7.2	Local Realism and Quantum Theory at Odds	194
10.8	Complementary Topics and Further Reading	196
10.9	Problems for Chapter 10	197

11 Working with Entanglement	199
11.1 Quantum Cryptography	199
11.1.1 The Vernam Coding	199
11.1.2 The B92 Protocol	200
11.1.3 EPR Protocols	202
11.1.4 The Scheme of Quantum Cryptography	203
11.2 One Qubit Transmits Two Bits (Dense Coding)	204
11.3 Quantum Teleportation	204
11.4 Entanglement Swapping	207
11.5 Spooky Action into the Past?*	208
11.6 Entanglement Distillation	209
11.7 A Measure of Entanglement for Mixtures: Entanglement of Formation and Concurrence*	212
11.8 Complementary Topics and Further Reading	216
11.9 Problems for Chapter 11	217
12 The Quantum Computer	219
12.1 Registers and Networks	219
12.2 Functional Computation	221
12.3 Quantum Parallelism	224
12.4 Two Simple Quantum Algorithms	225
12.4.1 The Deutsch Problem	225
12.4.2 The Deutsch-Jozsa Problem	226
12.5 Grover's Search Algorithm	228
12.6 Shor's Factorisation Algorithm	230
12.6.1 Reduction of Factorisation to the Search for a Period	231
12.6.2 The Quantum Algorithm for Determining the Period	234
12.7 Quantum Error Correction Using Non-local Measurements	239
12.7.1 Bit Flip Errors	239
12.7.2 Phase Flip Errors	240
12.8 The Components of the Quantum Computer*	241
12.9 Complementary Topics and Further Reading	243
12.10 Problems for Chapter 12	244
13 Generalised Measurements, POVM	247
13.1 The Function of a Generalised Dynamics of Open Quantum Systems	247
13.1.1 Problems	247
13.1.2 A Simple Example	248
13.2 The Non-optimal Stern-Gerlach Experiment and Generalised Measurements	251
13.2.1 The Experimental Setup	251
13.2.2 An Example of a Generalised Measurement	253
13.2.3 Unsharp Measurements	255
13.3 Generalised Measurements	256
13.3.1 What is a Quantum Measurement?	256
13.3.2 Generalised Measurement Postulates	257

13.3.3	The Polar Decomposition of a Linear Operator	258
13.3.4	Minimal Measurements and POVM	260
13.3.5	Implementation of a Generalised Measurement by Unitary Transformation and Projection	261
13.3.6	Entanglement Distillation by means of Generalised Measurements* .	262
13.4	POVM Measurements	263
13.4.1	Measurement Probabilities and Positive Operators	263
13.4.2	A Composite Measurement as an Example of a POVM Measurement	264
13.4.3	Can One Distinguish Between Two States with Certainty by a Single POVM Measurement?	265
13.4.4	The Advantage of a POVM Measurement for Determining States .	267
13.4.5	An Informationally-Complete POVM*	268
13.4.6	Estimating the State Before the Measurement	269
13.5	Complementary Topics and Further Reading	270
13.6	Problems for Chapter 13	270
14	The General Evolution of an Open Quantum System and Special Quantum Channels	273
14.1	Quantum Operations and their Operator-Sum Decompositions	273
14.1.1	Quantum Operations	273
14.1.2	The Operator-Sum Decomposition of Quantum Operations	275
14.1.3	Simple Quantum Operations	276
14.1.4	The Ambiguity of the Operator-Sum Decomposition	277
14.2	The Master Equation	277
14.3	Completely General Selective Measurements and POVM	279
14.4	Quantum Channels	281
14.4.1	The Depolarising Channel	281
14.4.2	Quantum Jumps and Amplitude Damping Channels	282
14.4.3	An Entanglement-Breaking Channel *	283
14.5	The Scenario and the Rules of Quantum Theory Revisited	284
14.6	Complementary Topics and Further Reading	288
14.7	Problems for Chapter 14	288
15	Decoherence and Approaches to the Description of the Quantum Measurement Process	289
15.1	Channels which Produce Decoherence	289
15.1.1	The Phase Damping Channel	289
15.1.2	Scattering and Decoherence	291
15.1.3	The Phase Flip Channel	292
15.2	Environment-Induced Decoherence	293
15.2.1	The Formation of the Classical World	293
15.2.2	Schrödinger's Cat	296
15.3	The Quantum Measurement Process*	297
15.3.1	The Research Programme*	297
15.3.2	Pre-Measurement*	298

15.3.3 Entanglement with the Environment Fixes the Observable*	299
15.3.4 Entanglement with Many Degrees of Freedom of the Environment* .	300
15.4 Has the Problem of Measurements been Solved?*	303
15.5 The Many-Worlds Interpretation*	303
15.6 Complementary Topics and Further Reading	304
15.7 Problems for Chapter 15	306
16 Two Implementations of Quantum Operations*	307
16.1 The Operator-Sum Decomposition*	307
16.2 The Unitary Implementation of Quantum Operations*	310
16.3 Implementation of a Completely General Selective Measurement by Unitary Transformation and Projection*	311
16.4 Complementary Topics and Further Reading*	313
16.5 Problems for Chapter 16	313
References	315
Reference categories	315
Bibliography	316
Subject Index	331