

# CONTENTS

---

<b>ONE</b>	<b>THE SOURCES AND PROPAGATION OF ERRORS</b>	<b>3</b>
1.1	Mathematical Preliminaries	3
1.2	Computer Representation of Numbers	12
1.3	Sources of Error	14
1.4	Propagation of Errors	19
1.5	Stability in Numerical Analysis	26
	Discussion of the Literature	32
	Problems	34
<b>TWO</b>	<b>ROOTFINDING FOR NONLINEAR EQUATIONS</b>	<b>39</b>
2.1	Simple Enclosure Methods	42
2.2	The Secant Method	48
2.3	Newton's Method	52
2.4	A General Theory for One-Point Iteration Methods	58
2.5	Aitken Extrapolation for Linearly Convergent Sequences	65
2.6	Error Tests	68
2.7	The Numerical Evaluation of Multiple Roots	71
2.8	Brent's Rootfinding Algorithm	75
2.9	Roots of Polynomials	78
2.10	Muller's Method	85
2.11	Nonlinear Systems of Equations	88
2.12	Newton's Method for Nonlinear Systems	92
	Discussion of the Literature	95
	Problems	97

<b>THREE</b>	<b>INTERPOLATION THEORY</b>	<b>107</b>
3.1	Polynomial Interpolation Theory	107
3.2	Newton Divided Differences	114
3.3	Finite Differences and Table-Oriented Interpolation Formulas	123
3.4	Errors in Data and Forward Differences	129
3.5	Further Results on Interpolation Error	132
3.6	Hermite Interpolation	137
3.7	Piecewise Polynomial Interpolation	141
	Discussion of the Literature	150
	Problems	153
<b>FOUR</b>	<b>APPROXIMATION OF FUNCTIONS</b>	<b>161</b>
4.1	The Weierstrass Theorem and Taylor's Theorem	162
4.2	The Minimax Approximation Problem	165
4.3	The Least Squares Approximation Problem	168
4.4	Orthogonal Polynomials	171
4.5	The Least Squares Approximation Problem (continued)	180
4.6	Economization of Taylor Series	186
4.7	Minimax Approximations	190
4.8	Near-Minimax Approximations	194
4.9	The Remes Algorithm	201
	Discussion of the Literature	203
	Problems	205
<b>FIVE</b>	<b>NUMERICAL INTEGRATION</b>	<b>213</b>
5.1	The Trapezoidal Rule and Simpson's Rule	215
5.2	Newton-Cotes Integration Formulas	225
5.3	Gaussian Quadrature	231
5.4	Patterson's Method	243
5.5	Asymptotic Error Formulas and Their Application	248
5.6	Adaptive Numerical Integration	263
5.7	Singular Integrals	268
	Discussion of the Literature	277
	Problems	279
<b>SIX</b>	<b>NUMERICAL METHODS FOR DIFFERENTIAL EQUATIONS</b>	<b>289</b>
6.1	Existence, Uniqueness, and Stability Theory	292
6.2	Euler's Method	300
6.3	Multistep Methods	314
6.4	The Midpoint Method	320

6.5	The Trapezoidal Method	325
6.6	A Low-Order Predictor-Corrector Algorithm	330
6.7	Derivation of Higher-Order Multistep Methods	338
6.8	Convergence and Stability Theory for Multistep Methods	350
6.9	Single-Step and Runge-Kutta Methods	366
	Discussion of Literature	380
	Problems	384
<b>SEVEN</b>	<b>LINEAR ALGEBRA</b>	<b>393</b>
7.1	Vector Spaces, Matrices, and Linear Systems	393
7.2	Eigenvalues and Canonical Forms for Matrices	401
7.3	Vector and Matrix Norms	412
7.4	Convergence and Perturbation Theorems	421
	Discussion of Literature	427
	Problems	428
<b>EIGHT</b>	<b>NUMERICAL SOLUTION OF SYSTEMS OF LINEAR EQUATIONS</b>	<b>435</b>
8.1	Gaussian Elimination	436
8.2	Pivoting and Scaling in Gaussian Elimination	444
8.3	Variants of Gaussian Elimination	450
8.4	Error Analysis	457
8.5	The Residual Correction Method	467
8.6	Iteration Methods	471
8.7	Error Prediction and Acceleration	478
8.8	The Numerical Solution of Poisson's Equation	482
	Discussion of Literature	487
	Problems	490
<b>NINE</b>	<b>THE MATRIX EIGENVALUE PROBLEM</b>	<b>499</b>
9.1	Eigenvalue Location, Error, and Stability Results	500
9.2	The Power Method	514
9.3	Orthogonal Transformations Using Householder Matrices	521
9.4	The Eigenvalues of a Symmetric Tridiagonal Matrix	532
9.5	The $QR$ Method	539
9.6	The Calculation of Eigenvectors and Inverse Iteration	548
	Discussion of Literature	553
	Problems	551
	<b>ANSWERS TO SELECTED EXERCISES</b>	<b>561</b>
	<b>INDEX</b>	<b>577</b>