

Contents

Preface	v
List of symbols and abbreviations	xi
1. Introduction	
1.1 The necessity for numerical quadrature and cubature	1
1.2 Applications of quadratures and cubatures	3
1.3 Historical remarks	7
1.4 Concept and aims of this book	9
1.5 General conventions	10
1.6 Remarks, problems, references	14
1.7 List of books concerning numerical quadrature and cubature	16
2. Construction Principles for Quadrature and Cubature Formulae	
2.1 Geometric constructions	18
2.2 The method of Taylor expansions	23
2.3 The method of undetermined coefficients	47
2.4 The method of generating functions	56
2.5 Interpolatory quadratures and cubatures	59
2.6 Composite rules	70
2.7 Monte Carlo and number theoretic methods	78
2.8 Remarks, problems, references	82
3. Error Analysis for Quadrature and Cubature Formulae	
3.1 Error representations for quadratures	93
3.2 Error representation for cubature formulae by Sard kernels	100
3.3 Determination and minimisation of error constants using Peano and Sard kernels	108

3.4	Further determination and minimisation of error constants	116
3.5	Error bounds without derivatives for quadratures on analytic function spaces	124
3.6	Error bounds without derivatives for quadratures and cubatures on spaces of functions which are analytic in a strip	142
3.7	Reducing errors	153
3.8	Remarks, problems, references	164
4.	Convergence of Quadrature and Cubature Procedures	
4.1	Definitions	170
4.2	How to prove convergence of quadrature and cubature procedures	172
4.3	Convergence conditions for general quadrature procedures	180
4.4	Convergence conditions for cubature procedures	190
4.5	Convergence for procedures consisting of composite rules	194
4.6	Remarks, references	197
5.	Orthogonal Polynomials	
5.1	Definitions and properties	199
5.2	Special systems of orthogonal polynomials	208
5.3	Implicitly-defined orthogonal polynomials	214
5.4	Numerical calculation of the zeros of orthogonal polynomials	226
5.5	Orthogonal polynomials in the complex plane	237
5.6	<i>N</i> -Dimensional orthogonal polynomials	239
5.7	Applications to cubature problems	246
5.8	Remarks, problems, references	260
6.	Interpolatory Quadrature and Cubature Formulae – Preassigned Nodes or Weights	
6.1	Classification and general properties	265
6.2	The Newton–Cotes quadrature formulae	267
6.3	Further quadratures with equidistant nodes	276
6.4	Newton–Cotes cubature formulae	277
6.5	The existence of positive cubature formulae with given nodes	286
6.6	Equally-weighted quadrature formulae – Tschebyscheff's problem	290

6.7	Equally-weighted cubature formulae	294
6.8	Remarks, problems, references	298
7.	Refined Interpolatory Quadrature	
7.1	General Gauss quadratures with free nodes	302
7.2	Gauss quadratures with preassigned nodes	305
7.3	Classical Gauss quadratures	315
7.4	Modifications of classical Gauss quadratures having some preassigned nodes	321
7.5	Quadratures with minimal error norm	335
7.6	Rectangle and trapezoidal rules	342
7.7	Approximate computation of indefinite integrals	344
7.8	Remarks, problems, references	366
8.	Non-interpolatory Quadratures	
8.1	Richardson extrapolation	369
8.2	Convergence of the extrapolation	375
8.3	Properties of Romberg's quadrature	381
8.4	Romberg cubature	391
8.5	Numerical integration of rapidly-oscillating functions	397
8.6	Automatic integration – Adaptive methods	401
8.7	Remarks, problems, references	403
9.	Auxiliary Material	
9.1	Bibliographic material	408
9.2	Testing quadrature subroutines	409
9.3	Some published programs	415
9.4	A list of tables	419
Author Index	429	
Index of Key-words	433	
Index of Programs and Tables	441	