

Contents

1.	A Survey of Boundary Integral Equation Methods for the Numerical Solution of Laplace's Equation in Three Dimensions	1
<i>K. E. Atkinson</i>		
1.	Introduction	1
2.	Integral Equation Reformulations of Laplace's Equation	2
2.1.	Direct BIE Methods	4
2.2.	Indirect BIE Methods	5
2.3.	The Smooth Boundary Case	7
2.4.	Piecewise Smooth Boundaries	8
2.5.	The Poisson Equation	9
3.	Numerical Methods for Boundary Integral Equations	9
3.1.	Global Methods	11
4.	Boundary Element Methods	14
4.1.	Strongly Elliptic Operator Framework	14
4.2.	Collocation Methods	18
5.	Numerical Integration	21
5.1.	Boundary Element Integrations	22
6.	Iterative Methods of Solution	25
6.1.	A Two-Grid Method	26
	References	29
2.	Superconvergence	35
<i>I. H. Sloan</i>		
1.	Introduction	35
2.	Galerkin and Collocation Methods	38
3.	Iterated Projection Method	40
3.1.	Iterated Kantorovich Method	45
3.2.	An Example	47
4.	Linear Functionals of the Galerkin Approximation	48
4.1.	Approximation of (y, g) by (y_h, g)	49
4.2.	The Iterated Galerkin Method Revisited	51
4.3.	HMP Method for Approximating (y, g)	53
4.4.	Discrete Galerkin and Iterated Galerkin Methods	55
5.	Iterated Collocation Method	56
5.1.	Iterated Collocation for Piecewise-Polynomial Spaces S_h	58
5.2.	Collocation at the Gauss Points	59

5.3. Iterated Collocation Method versus Iterated Galerkin Method	63
5.4. Discrete Collocation and Iterated Collocation Methods	64
6. Nonlinear Integral Equations	64
7. Concluding Remarks	67
Acknowledgments	67
References	67

3. Perturbed Projection Methods for Various Classes of Operator and Integral Equations

M. A. Golberg

1. Introduction	71
2. Projection Methods and Their Variants	73
2.1. Notation	74
2.2. Projection Methods	75
2.3. Convergence Analysis of Projection Methods	81
2.4. Some Variants of the Projection Method	84
3. Perturbed Projection Methods	91
3.1. The Perturbed Sloan Iterate	92
3.2. The Perturbed Kantorovich Method	93
3.3. The Perturbed Kantorovich Iterate	94
3.4. The Perturbed Dellwo-Friedman Method	95
4. Galerkin's Method with Quadrature Errors	95
5. Direct Analysis of Galerkin's Method with Quadrature Errors	106
6. Collocation Methods with Quadrature Errors	110
6.1. Collocation Method	110
6.2. Direct Analysis of the Discrete Collocation Method	113
6.3. Superconvergence of Collocation for Volterra Equations	117
7. Galerkin's Method for Equations with Positive-Definite Dominant Parts	120
8. Conclusions	126
References	127

4. Numerical Solution of Parallel Processors of Two-Point Boundary-Value Problems of Astrodynamics

G. Miel

1. Introduction	131
2. Mathematical Preliminaries	134
2.1. The Integral Equation	135
2.2. The Legendre Polynomials	136
2.3. The Hilbert Product Space	138
2.4. Operator Equations	140
2.5. The Newton-Kantorovich Method	141
3. Equations of Motion	144
3.1. Keplerian Motion	147
3.2. Aspherical Gravitational Potential	147
3.3. Earth Satellite Orbits	148
3.4. Perturbations of GPS Orbits	149

4. The Well-Posedness Issue	149
4.1. Example	150
4.2. Method of Patched Conics	151
4.3. Grand Tour of Voyager 2	152
4.4. Picard Iteration	154
4.5. Example: Perturbed Keplerian Motion	155
4.6. Newton-Kantorovich Iteration	157
5. Perturbed Galerkin Method	158
5.1. Equivalence	161
5.2. Analytic Principle	163
5.3. Convergence Result	164
6. Parallel Algorithms	165
6.1. Setup of the Matrix	167
6.2. Setup of the Constant Vector	168
6.3. Solution of the Linear System	170
6.4. Legendre Polynomial Expansion	171
6.5. Odds and Ends	172
7. Numerical Examples	174
7.1. Earth-Mars Trajectories	174
7.2. Trajectory Optimization	177
8. Conclusion	179
References	179

5. Introduction to the Numerical Solution of Cauchy Singular Integral Equations 183

M. A. Golberg

1. Introduction	183
2. Analytical Solution of the Airfoil Equation	185
3. Determining c	190
3.1. The Kutta Condition	191
3.2. $\int_{-1}^1 v(t) dt = M$	192
3.3. $v(1) = v(-1) = 0$	192
4. Numerical Methods for the Generalized Airfoil Equation	193
5. Indirect Methods	194
6. Direct Methods	197
7. Some Mapping Properties of the Airfoil Operator	198
8. Operator Formulation of the Generalized Airfoil Equation	204
9. Degenerate Kernel Methods	207
10. Galerkin's Method	209
10.1. Galerkin's Method: $\nu = 0$	210
10.2. A Superconvergence Result for $\nu = 0$	214
10.3. Galerkin's Method: $\nu = 1$	218
10.4. Galerkin's Method: $\nu = 1$	220
11. Collocation	220
11.1. $\nu = 0$: Continuous Data	221
11.2. $\nu = 0$: $k(x, t) = a(x, t) \log(x - t) + b(x, t)$	222
11.3. Polynomial Collocation: $\nu = \pm 1$	228

12. Quadrature Methods	230
12.1. Quadrature Rules for Cauchy Principal-Value Integrals	230
12.2. The Gaussian Quadrature Method	233
12.3. Lobatto Quadrature: $\nu = 1$	236
12.4. Endpoint Convergence of the Lobatto Quadrature Method	238
12.5. Lobatto Quadrature: $\nu = 0$	240
13. Kantorovich Regularization	241
14. Product Quadrature	246
15. Kalandiya's Method	247
16. Some Other Numerical Methods for the GAE	249
16.1. Conversion to a Logarithmic Equation	250
16.2. Some Other Polynomial Methods for Solving the GAE	255
17. Numerical Solution of CSIES of the Second Kind with Constant Coefficients	257
17.1. The Standard Polynomial Algorithms	258
18. Other Polynomial Approximation Methods	268
18.1. Cohen's Method	269
18.2. The Method of Chawla and Kumar	270
18.3. Hashmi and Delves' Algorithm	272
18.4. Piecewise Polynomial Methods	272
19. Convergence	272
19.1. Mean-Square Convergence of Galerkin's Method: $\nu = 0$	273
19.2. Mean-Square Convergence of Galerkin's Method: $\nu = 1$	278
19.3. Uniform Convergence of Galerkin's Method: $\nu = 0, 1$	281
19.4. The Sloan Iterate	283
19.5. Convergence of the Collocation Method	284
19.6. Convergence of the Discrete Galerkin Method	287
19.7. Convergence of the Gaussian Quadrature Method	293
20. Conclusions	296
References	296
6. Convergence Theorems for Singular Integral Equations	309
<i>D. Elliott</i>	
1. Introduction	309
2. A Theory for the Singular Integral Equation	310
3. The Approximate Solution of Singular Integral Equations	322
3.1. An Indirect Method	323
4. Direct Methods and Analysis of their Convergence	328
5. Some Examples of Direct Methods	342
5.1. The Galerkin-Petrov Method	342
5.2. A Collocation Method	348
5.3. A Discrete Galerkin Method	356
6. Conclusions	359
Acknowledgment	359
References	359
7. Planing Surfaces	363
<i>E. O. Tuck</i>	
1. Introduction	363
2. The Planing Equation	364

3. Generalizations	368
References	371
8. Abel Integral Equations	373
<i>R. S. Anderssen and F. R. de Hoog</i>	
1. Introduction	373
2. Abel Integral Equations in Applications	376
2.1. First-Kind Abel Integral Equations in Geometric Probability	377
2.2. First-Kind Abel Integral Equations in Interferometry	377
2.3. First-Kind Abel Integral Equations in Stereology	378
2.4. Second-Kind Abel Integral Equations in Stereology	379
2.5. The Abel Integral Equation of Seismology	379
2.6. Abel Integral Equations in Tomography	380
3. The Numerical Analysis of Abel Integral Equations	382
4. Pseudoanalytic Methods	387
5. Wiener Filtering	394
6. Stabilized Evaluation of Inversion Formulas	400
7. The Data Functional Strategy	402
8. Choice of Algorithm	403
References	406
<i>Index</i>	411