
Contents

Chapter 1. Auxiliary material	15
Operators	15
Operator matrices and their determinants	18
Banach algebras	19
C^* -algebras	21
Local principles.	22
Convergence manifold and error estimates	25
Stability in the sense of Mikhlin	30
The Aubin-Nitsche lemma	31
Notes and comments	38
Chapter 2. Approximation theory	39
Part A. Spline approximation in periodic Sobolev spaces	39
Periodic Sobolev spaces	39
Approximation and inverse properties of splines	43
Commutator property of splines	48
The Arnold-Wendland lemma.	52
Periodic spline interpolation	56
Part B. Trigonometric approximation	67
Generalized Hölder-Zygmund spaces.	67
Approximation by Fourier sums	70
Best approximation and operator norms	74
Trigonometric interpolation	77
Notes and comments	79
Chapter 3. Approximation methods for Fredholm integral equations	80
Polynomial approximation of Fredholm integral equations	80
The numerical solution of first-kind integral equations with logarithmic kernels	86
Spline approximation of Fredholm integral equations	89
Notes and comments	93
Chapter 4. Functions of shifts on Banach spaces and their finite sections	94
Continuous functions of shifts	94
Examples of V -dominating algebras	101
The algebra $\text{alg}(V, V_{-1})$	103
Decomposing algebras $\text{alg}(V, V_{-1})$	109

Finite sections	111
Convergence of the finite section method	113
\mathcal{P} -convergent operator sequences	114
Non-strongly converging approximation methods	118
Stability of finite sections	119
Toeplitz operators	120
Wiener-Hopf integral operators	128
Modified finite sections	135
Abstract paired operators	138
Paired discrete Wiener-Hopf operators	144
Paired Wiener-Hopf integral operators	147
Singular integral operators on spaces $L^p(\mathbf{T}, \varrho)$	151
Systems	153
Block Toeplitz operators	157
Systems of Wiener-Hopf integral equations	159
Systems of singular integral equations	162
Singular integral operators on the interval $(-1, 1)$	162
Notes and comments	166
Chapter 5. Spline approximation methods for classes of convolution equations	167
A class of non-compact integral operators	168
Smoothness of solutions	172
Meshes	174
Spline spaces and projections	176
Galerkin methods with piecewise polynomials: Wiener-Hopf equations	182
Galerkin methods with piecewise polynomials: Mellin convolution equations	186
Collocation with piecewise polynomials: Mellin convolution equations	192
Collocation with piecewise polynomials: Wiener-Hopf equations	196
Collocation with continuous piecewise polynomials: Mellin convolution equations	197
Collocation with continuous piecewise polynomials: Wiener-Hopf equations	202
Discrete methods: Nyström methods and discrete collocation for Mellin convolution equations	204
Discrete methods: Nyström methods and discrete collocation for Wiener-Hopf equations	208
Systems	211
Notes and comments	213
Chapter 6. Singular integral equations and pseudodifferential equations on curves	215
Curves and singular integral operators in L^2 -spaces	215
The algebra $\hat{\Sigma}^k(\alpha)$	218
The algebra $\text{alg}\{S_T, \text{PC}(I)\}$ and symbols	219
Toeplitz operators	227
Strong ellipticity	229
Singular integral operators on Hölder-Zygmund spaces	236
Smoothness and asymptotics of solutions	241
Pseudodifferential operators on a closed curve	248
Notes and comments	260
Chapter 7. Polynomial approximation methods for the solution of singular integral equations with piecewise continuous coefficients on the unit circle	262
The approximation methods	262
Algebraization	266

The finite section method	270
The collocation method	279
Systems	282
Subalgebras	284
A generalization	285
Notes and comments	288
Chapter 8. Polynomial approximation methods for singular integral equations in Zygmund-Hölder spaces	289
Factorization	289
Finite sections	290
The collocation method	294
Mechanical quadratures	296
Notes and comments	299
Chapter 9. Polynomial approximation methods for singular integral equations on intervals.	300
Singular integrals and piecewise holomorphic functions	300
Special mapping properties	304
Fredholm properties	310
Quadrature rules	312
Convergence analysis	316
Zero distribution	322
Numerical construction of Gauss type quadrature rules	325
An algorithm for the case $\kappa = -1$	330
The special case of singular integral operators with constant real coefficients	333
Some further results	334
Notes and comments	341
Chapter 10. Spline approximation and quadrature methods for the solution of singular integral equations on closed curves	343
Preliminaries	343
Collocation methods	346
Galerkin methods	350
Quadrature methods in the case of the unit circle	352
Error estimates	361
Quadrature methods in the case of an arbitrary closed curve	366
Error estimates	370
Banach algebra approach to the stability of paired circulants	373
Notes and comments	386
Chapter 11. Mellin techniques in the numerical analysis of singular integral equations	387
Quadrature methods for singular integral equations on curves with corner points	390
Collocation methods for singular integral equations on curves with corners. Piecewise constant trial functions	410
Quadrature methods for singular integral equations on the interval	413
Quadrature methods for Mellin operators of order zero	426

Chapter 12. Spline approximation and quadrature methods for singular integral equations on an interval. Graded meshes	439
Galerkin methods	439
Collocation methods.	447
Region method with splines of even degree.	453
Quadrature methods for strongly elliptic Cauchy singular integral equations on an interval.	455
A quadrature method for a Cauchy singular integral equation with a fixed singularity	476
Notes and comments	485
Chapter 13. Spline approximation methods for pseudodifferential equations on closed curves	486
Spline collocation on uniform meshes	486
Spline collocation on arbitrary meshes	503
Spline Galerkin method on arbitrary meshes	508
Systems of pseudodifferential equations	510
Notes and comments	511
References	513
Notation index	535
Name index	537
Subject index	540