

CONTENTS

Chapter 1	Introductory Ideas	1
1.1	A Simply Stated Problem, 1	
1.2	Linear Spaces, 2	
1.3	Normed Linear Spaces, 9	
1.4	The Space $L_2[a, b]$, 11	
1.5	Basis for a Linear Space, 15	
1.6	Approximating from Finite Dimensional Subspaces, 19	
Chapter 2	Lagrangian Interpolates	24
2.1	Introduction, 24	
2.2	On Polynomials, 26	
2.3	Lagrange Interpolation, 29	
2.4	Computation and Choice of Basis, 32	
2.5	Error Estimates for Lagrange Interpolates, 35	
2.6	Best Approximation and Extended Error Estimates, 40	
2.7	Piecewise Lagrange Interpolation, 44	
Chapter 3	Hermitian Interpolates	52
3.1	Introduction, 52	
3.2	Computation of Piecewise Cubic Hermites, 56	
3.3	A Simple Application, 60	
3.4	Hermite Interpolation, 63	
3.5	Piecewise Hermite Interpolation, 68	
3.6	Computation of Piecewise Hermite Polynomials, 70	
3.7	The Hermite-Birkhoff Interpolation Problem, 74	
Chapter 4	Polynomial Splines and Generalizations	77
4.1	Introduction, 77	
4.2	Cubic Splines, 78	
4.3	Derivation of the B Splines, 87	

4.4	Splines and Ordinary Differential Equations, 94	
4.5	Error Analysis, 107	
Chapter 5	Approximating Functions of Several Variables	116
5.1	Surface Fitting, 116	
5.2	Approximates on a Rectangular Grid, 118	
5.3	Tensor Products, 135	
5.4	Approximates on Triangular Grids, 137	
5.5	Automatic Mesh Generation and Isoparametric Transforms, 155	
5.6	Blended Interpolates and Surface Approximation, 168	
Chapter 6	Fundamentals for Variational Methods	174
6.1	Variational Methods, 174	
6.2	Linear Operators, 177	
6.3	Inner Product Spaces, 182	
6.4	Norms, Convergence, and Completeness, 187	
6.5	Equivalent Norms, 190	
6.6	Best Approximations, 192	
6.7	Least Squares Fits, 197	
Chapter 7	The Finite Element Method	201
7.1	Introduction, 201	
7.2	A Simple Application, 205	
7.3	An Elementary Error Analysis, 211	
7.4	Lowering the Smoothness Requirements—Choice of Linear Space, 217	
7.5	Some Practical Considerations, 225	
7.6	Applications to the Dirichlet Problem, 227	
7.7	The Mixed Boundary Value Problem, 240	
7.8	The Neumann Problem, 245	
7.9	Coerciveness and Rates of Convergence, 251	
7.10	Curved Boundaries and Nonconforming Elements, 255	
7.11	Higher Order Linear Ordinary Differential Equations, 257	
7.12	Second and Higher Order Elliptic Partial Differential Equations, 262	
7.13	Galerkin Methods and Least Squares Methods, 267	
Chapter 8	The Method of Collocation	273
8.1	Introduction, 273	
8.2	A Simple Special Case: Existence Via Matrix Analysis, 279	

- 8.3 Green's Functions, 286
- 8.4 Collocation Existence Via Green's Functions, 289
- 8.5 Error Analyses Via Green's Functions, 296
- 8.6 Collocation and Partial Differential Equations, 298
- 8.7 Orthogonal Collocation, 304
- 8.8 A Connection Between Collocation and Galerkin Methods, 314

Glossary of Symbols **319**

Index **321**