

Contents

| | |
|-------------------------------|---|
| Introduction | 1 |
| Principal Notations | 4 |

Chapter I

Minimization of Functions and Unilateral Boundary Value Problems

| | |
|---|----|
| 1. Minimization of Coercive Forms | 6 |
| 1.1. Notation | 6 |
| 1.2. The Case when π is Coercive | 6 |
| 1.3. Characterization of the Minimizing Element. Variational Inequalities | 9 |
| 1.4. Alternative Form of Variational Inequalities | 11 |
| 1.5. Function J being the Sum of a Differentiable and Non-Differentiable Function | 12 |
| 1.6. The Convexity Hypothesis on \mathcal{U}_{ad} | 13 |
| 1.7. Orientation | 15 |
| 2. A Direct Solution of Certain Variational Inequalities | 15 |
| 2.1. Problem Statement | 15 |
| 2.2. An Existence and Uniqueness Theorem | 15 |
| 3. Examples | 18 |
| 3.1. Function Spaces on Ω | 18 |
| 3.2. Function Spaces on Γ | 19 |
| 3.3. Subspaces of $H^m(\Omega)$ | 20 |
| 3.4. Examples of Boundary Value Problems | 22 |
| 3.5. Unilateral Boundary Value Problems (I) | 27 |
| 3.6. Unilateral Boundary Value Problems (II) | 29 |
| 3.7. Unilateral Boundary Value Problems (III) | 30 |
| 3.8. Unilateral Boundary Value Problems; Case of Systems | 31 |
| 3.9. Elliptic Operators of Order Greater than Two | 32 |
| 3.10. Non-differentiable Functionals | 34 |
| 4. A Comparison Theorem | 36 |
| 4.1. General Results | 36 |
| 4.2. An Application | 37 |

| | |
|--|-----|
| 4.5. Passage to the Limit | 145 |
| 4.6. Integro-Differential Equation of Riccati Type | 146 |
| 4.7. Connections with the Hamilton-Jacobi Theory | 150 |
| 4.8. The Case where Constraints are Present | 152 |
| 4.9. Various Remarks | 153 |
| 4.9.1. Direct Study of the "Riccati Equation" | 153 |
| 4.9.2. Another Approach to the Direct Study of the "Riccati Equation" . | 155 |
| 4.9.3. Yet Another Approach to the Direct Study of the "Riccati Equation" . | 155 |
| 5. Decoupling and Integro-Differential Equation of Riccati Type (II) | 157 |
| 5.1. Application of the Schwartz-Kernel Theorem | 157 |
| 5.2. Example of a Mixed Neumann Problem with Boundary Control . | 159 |
| 5.3. Example of a Mixed Neumann Problem with Observation of the Final State | 163 |
| 5.4. Mixed Neumann Problem, Observation of the Final State and Constraints in a Vector Space | 166 |
| 5.5. Remarks on Decoupling in the Presence of Constraints | 166 |
| 6. Behaviour as $T \rightarrow +\infty$ | 167 |
| 6.1. Orientation and Hypotheses | 167 |
| 6.2. The Case $T = \infty$ | 168 |
| 6.3. Passage to the Limit as $T \rightarrow +\infty$ | 172 |
| 7. Problems which are not Necessarily Coercive. | 175 |
| 7.1. Distributed Observation | 175 |
| 7.2. Observation of the Final State | 176 |
| 7.3. Examples where $N=0$ and \mathcal{U}_{ad} is not Bounded | 180 |
| 8. Other Observations of the State and other Types of Control | 182 |
| 8.1. Pointwise Observation of the State | 182 |
| 8.2. Pointwise Control | 186 |
| 8.3. Control and Observation on the Boundary | 187 |
| 9. Boundary Control and Observation on the Boundary or of the Final State for a System Governed by a Mixed Dirichlet Problem | 193 |
| 9.1. Orientation and Problem Statement | 193 |
| 9.2. Non Homogeneous Mixed Dirichlet Problem | 194 |
| 9.3. Definition of $\frac{\partial y}{\partial v_A}$; Observation | 197 |
| 9.4. Cost Function; Equations of Optimal Control | 198 |
| 9.5. Regular Control | 201 |
| 9.6. Observation of the Final State | 202 |
| 9.7. Observation of the Final State, Second Order Parabolic Operator . | 205 |
| 10. Controllability | 207 |
| 10.1. Problem Statement | 207 |
| 10.2. Controllability and Uniqueness | 207 |
| 10.3. Super-Controllability and Super-Uniqueness | 211 |

| | | |
|-----------------|---|-----|
| 11. | Control via Initial Conditions; Estimation | 213 |
| 11.1. | Problem Statement. General Results | 213 |
| 11.2. | Examples | 214 |
| 11.3. | Controllability | 215 |
| 11.4. | An Estimation Problem | 216 |
| 12. | Duality | 219 |
| 12.1. | General Remarks | 219 |
| 12.2. | Example | 222 |
| 13. | Constraints on the Control and the State | 223 |
| 13.1. | A General Result | 223 |
| 13.2. | Applications (I) | 226 |
| 13.3. | Applications (II) | 228 |
| 14. | Non Quadratic Cost Functions | 230 |
| 14.1. | Orientation | 230 |
| 14.2. | An Example | 230 |
| 14.3. | Remarks on Decoupling | 233 |
| 15. | Existence Results for Optimal Controls | 235 |
| 15.1. | Orientation | 235 |
| 15.2. | Non-linear Problem with Distributed Control (I) | 235 |
| 15.3. | Non-linear Problem with Distributed Control. Singular Perturbation | 237 |
| 15.4. | Non-linear Problem. Boundary Control | 241 |
| 15.5. | Utilization of Convexity and the Maximum Principle for Second Order Parabolic Equations | 243 |
| 15.6. | Control of Systems Governed by Evolution Inequalities | 247 |
| 16. | First Order Necessary Conditions | 248 |
| 16.1. | Statement of the Theorem | 248 |
| 16.2. | Proof of Theorem 16.1 | 249 |
| 16.2.1. | “Algebraic” Transformation | 249 |
| 16.2.2. | Utilization of (16.11.) | 250 |
| 16.2.3. | Proof of (16.12.) | 250 |
| 16.3. | Remarks | 251 |
| 17. | Time Optimal Control | 252 |
| 17.1. | Problem Statement | 252 |
| 17.2. | Existence Theorem | 253 |
| 17.3. | Bang-Bang Theorem | 254 |
| 18. | Miscellaneous | 262 |
| 18.1. | Equations with Delay | 262 |
| 18.1.1. | Definition of the State | 262 |
| 18.1.2. | Control Problem | 263 |
| 18.2. | Spaces which are not Normable | 264 |
| Notes | | 266 |

Chapter IV

Control of Systems Governed by Hyperbolic Equations or by Equations which are well Posed in the Petrowsky Sense

| | | |
|------|---|-----|
| 1. | Second Order Evolution Equations | 272 |
| 1.1. | Notation and Hypotheses | 272 |
| 1.2. | Problem Statement. An Existence and Uniqueness Result | 273 |
| 1.3. | Proof of Uniqueness | 274 |
| 1.4. | Proof of Existence | 275 |
| 1.5. | Examples (I) | 278 |
| 1.6. | Examples (II) | 280 |
| 1.7. | Orientation | 281 |
| 2. | Control Problems | 281 |
| 2.1. | Notation. Immediate Properties | 281 |
| 2.2. | Case (2.5.) | 282 |
| 2.3. | Case (2.6.) | 284 |
| 2.4. | Case (2.7.) | 289 |
| 2.5. | Case (2.8.) | 291 |
| 3. | Transposition and Applications to Control | 291 |
| 3.1. | Transposition of Theorem 1.1. | 291 |
| 3.2. | Application (I) | 293 |
| 3.3. | Application (II) | 294 |
| 3.4. | Application (III) | 294 |
| 4. | Examples | 295 |
| 4.1. | Examples of Hyperbolic Problems. Distributed Control, Distributed Observation | 295 |
| 4.2. | Examples of Hyperbolic Systems. Distributed Control, Observation of the Final State | 298 |
| 4.3. | Petrowsky Type Equation. Distributed Control. Distributed Observation | 299 |
| 4.4. | Petrowsky Type Equation. Distributed Control. Observation of the Final State | 300 |
| 4.5. | Orientation | 301 |
| 5. | Decoupling | 301 |
| 5.1. | Problem Statement. Rewriting as a System of First Order Equations . | 301 |
| 5.2. | Rewriting of the Set of Equations Determining the Optimal Control. . | 303 |
| 5.3. | Decoupling | 306 |
| 5.4. | Riccati Integro-differential Equation | 310 |
| 5.5. | Another Optimal Control Problem. Decoupling | 312 |
| 6. | Control via Initial Conditions. Estimation | 314 |
| 6.1. | Problem Statement | 314 |
| 6.2. | Coercivity of $J(v)$ | 315 |
| 6.3. | System of Equations Determining the Optimal Control | 316 |

| | |
|---|-----|
| 7. Boundary Control (I) | 318 |
| 7.1. Problem Statement | 318 |
| 7.2. Definition of the State of the System | 319 |
| 7.3. Distributed Observation | 321 |
| 7.4. Boundary Observation | 322 |
| 8. Boundary Control (II). | 325 |
| 8.1. Problem Statement | 325 |
| 8.2. Control v Regular | 325 |
| 8.3. Examples | 327 |
| 9. Parabolic-Hyperbolic Systems | 328 |
| 9.1. Recapitulation of Some General Results | 328 |
| 9.2. Complement | 331 |
| 9.3. Control Problems | 333 |
| 9.4. Example (I) | 335 |
| 9.5. Example (II) | 336 |
| 9.6. Decoupling | 337 |
| 10. Existence Theorems | 341 |
| 10.1. Orientation | 341 |
| 10.2. Example. Introduction of a "Viscosity" Term | 342 |
| 10.3. Time Optimal Control | 344 |
| Notes | 347 |

Chapter V

Regularization, Approximation and Penalization

| | |
|---|-----|
| 1. Regularization | 350 |
| 1.1. Parabolic Regularization | 350 |
| 1.2. Application to Optimal Control | 353 |
| 1.3. Application to Decoupling | 357 |
| 1.4. Various Remarks | 359 |
| 1.5. Regularization of the Control | 360 |
| 2. Approximation in Terms of Systems of Cauchy-Kowaleska Type | 364 |
| 2.1. Evolution Equation on a Variety | 364 |
| 2.2. Approximation by a System of Cauchy-Kowaleska Type | 370 |
| 2.3. Linearized Navier-Stokes Equation | 375 |
| 3. Penalization | 377 |
| Notes | 383 |
| Bibliography | 384 |
| Subject Index | 395 |