

# CONTENTS

Introduction	vii
<b>Chapter 1. What Is Control Theory?</b>	
1.1. Introduction	1
1.2. Systems	2
1.3. Schematics	2
1.4. Mathematical Systems	4
1.5. The Behavior of Systems	4
1.6. Improvement of the Behavior of Systems	5
1.7. More Detailed Breakdown	6
1.8. Uncertainty	7
1.9. Conclusion	7
Bibliography and Comments	8
<b>Chapter 2. Second-Order Linear Differential and Difference Equations</b>	
2.1. Introduction	10
2.2. Second-Order Linear Differential Equations with Constant Coefficients	10
2.3. The Inhomogeneous Equation	12
2.4. Two-Point Boundary Conditions	13
2.5. First-Order Linear Differential Equations with Variable Coefficients	14
2.6. The Riccati Equation	15
2.7. Linear Equations with Variable Coefficients	17
2.8. The Inhomogeneous Equation	18
2.9. Green's Function	20
2.10. Linear Systems	22
2.11. Difference Equations	23

Miscellaneous Exercises	25
Bibliography and Comments	27

### Chapter 3. **Stability and Control**

3.1. Introduction	28
3.2. Stability	30
3.3. Numerical Solution and Stability	33
3.4. Perturbation Procedures	34
3.5. A Fundamental Stability Theorem	36
3.6. Stability by Design	37
3.7. Stability by Control	37
3.8. Proportional Control	39
3.9. Discussion	40
3.10. Analytic Formulation	41
3.11. One-Dimensional Systems	42
Miscellaneous Exercises	44
Bibliography and Comments	46

### Chapter 4. **Continuous Variational Processes: Calculus of Variations**

4.1. Introduction	49
4.2. Does a Minimum Exist?	50
4.3. The Euler Equation	52
4.4. A Fallacious Argument	54
4.5. Haar's Device	54
4.6. Solution of the Euler Equation	55
4.7. Minimizing Property of the Solution	56
4.8. Alternative Approach	57
4.9. Asymptotic Control	57
4.10. Infinite Control Process	59
4.11. The Minimum Value of $J(u)$	60
4.12. Two-Point Constraints	62
4.13. Terminal Control	63
4.14. The Courant Parameter	65
4.15. Successive Approximations	65
4.16. $\min \int_0^1 [u'^2 + g(t)u^2] dt$	67
4.17. Discussion	69
4.18. The Simplicity of Control Processes	69
4.19. Discussion	71
4.20. The Minimum Value of $J(u)$	72
4.21. A Smoothing Process	72

4.22. Variation-Diminishing Property of Green's Function	74
4.23. Constraints	76
4.24. Minimizing Property	77
4.25. Monotonicity in $\lambda$	77
4.26. Proof of Monotonicity	78
4.27. Discussion	79
4.28. More General Quadratic Variational Problems	79
4.29. Variational Procedure	80
4.30. Proof of Minimum Property	81
4.31. Existence and Uniqueness	82
4.32. The Adjoint Operator	82
4.33. Sturm-Liouville Theory	85
4.34. Minimization by Means of Inequalities	87
4.35. Multiple Constraints	88
4.36. Unknown External Forces	90
Miscellaneous Exercises	91
Bibliography and Comments	99

## Chapter 5. **Dynamic Programming**

5.1. Introduction	101
5.2. Control as a Multistage Decision Process	101
5.3. Preliminary Concepts	103
5.4. Formalism	104
5.5. Principle of Optimality	107
5.6. Discussion	108
5.7. Simplification	109
5.8. Validation	109
5.9. Infinite Process	110
5.10. Limiting Behavior as $T \rightarrow \infty$	110
5.11. Two-Point Boundary Problems	112
5.12. Time-Dependent Control Process	113
5.13. Global Constraints	114
5.14. Discrete Control Processes	116
5.15. Preliminaries	117
5.16. Recurrence Relation	117
5.17. Explicit Recurrence Relations	118
5.18. Behavior of $r_N$	119
5.19. Approach to Steady-State Behavior	120
5.20. Equivalent Linear Relations	120
5.21. Local Constraints	121
5.22. Continuous as Limit of Discrete	123
5.23. Bang-Bang Control	124
5.24. Control in the Presence of Unknown Influences	125

5.25. Comparison between Calculus of Variations and Dynamic Programming	126
Miscellaneous Exercises	127
Bibliography and Comments	135
<b>Chapter 6. Review of Matrix Theory and Linear Differential Equations</b>	
6.1. Introduction	138
6.2. Vector-Matrix Notation	139
6.3. Inverse Matrix	140
6.4. The Product of Two Matrices	141
6.5. Inner Product and Norms	143
6.6. Orthogonal Matrices	145
6.7. Canonical Representation	146
6.8. $\text{Det } A$	147
6.9. Functions of a Symmetric Matrix	147
6.10. Positive Definite Matrices	149
6.11. Representation of $A^{-1}$	150
6.12. Differentiation and Integration of Vectors and Matrices	151
6.13. The Matrix Exponential	152
6.14. Existence and Uniqueness Proof	152
6.15. Euler Technique and Asymptotic Behavior	156
6.16. $x'' - A(t)x = 0$	157
6.17. $x' = Ax + By, y' = Cx + Dy$	158
6.18. Matrix Riccati Equation	159
6.19. $dX/dt = AX + XB, X(0) = C$	159
Miscellaneous Exercises	160
Bibliography and Comments	164
<b>Chapter 7. Multidimensional Control Processes via the Calculus of Variations</b>	
7.1. Introduction	165
7.2. The Euler Equation	166
7.3. The Case of Constant $A$	167
7.4. Nonsingularity of $\cosh ST$	169
7.5. The Minimum Value	169
7.6. Asymptotic Behavior	170
7.7. Variable $A(t)$	171
7.8. The Nonsingularity of $X_2'(T)$	171
7.9. The Minimum Value	172
7.10. Computational Aspects	173

7.11. $\min \int_0^T [(x, x) + (y, y)] dt, x' = Bx + y, x(0) = c$	174
Miscellaneous Exercises	175

## Chapter 8. **Multidimensional Control Processes via Dynamic Programming**

8.1. Introduction	178
8.2. $\int_0^T [(x', x') + (x, Ax)] dt$	179
8.3. The Associated Riccati Equation	180
8.4. Asymptotic Behavior	181
8.5. Rigorous Aspects	182
8.6. Time-Dependent Case	182
8.7. Computational Aspects	183
8.8. Successive Approximations	184
8.9. Approximation in Policy Space	185
8.10. Monotone Convergence	186
8.11. Partitioning	187
8.12. Power Series Expansions	188
8.13. Extrapolation	189
8.14. Minimization via Inequalities	191
8.15. Discrete Control Processes	193
8.16. Ill-Conditioned Linear Systems	195
8.17. Lagrange Multipliers	198
8.18. Reduction of Dimensionality	199
8.19. Successive Approximations	202
8.20. Distributed Parameters	204
8.21. Slightly Intertwined Systems	205
Miscellaneous Exercises	208
Bibliography and Comments	214

## Chapter 9. **Functional Analysis**

9.1. Motivation	217
9.2. The Hilbert Space $L^2(0, T)$	218
9.3. Inner Products	219
9.4. Linear Operators	220
9.5. Vector Hilbert Space	220
9.6. Quadratic Functionals	221
9.7. Existence and Uniqueness of a Minimizing Function	222
9.8. The Equation for the Minimizing Function	224
9.9. Application to Differential Equations	226
9.10. Numerical Aspects	227
9.11. A Simple Algebraic Example	228

9.12. The Equation $x + \lambda Bx = c$	229
9.13. The Integral Equation $f(t) + \lambda \int_0^T K(t, t_1) dt_1 = g(t)$	231
9.14. Lagrange Multipliers	231
9.15. The Operator $R_a$	232
9.16. Control Subject to Constraints	234
9.17. Properties of $\varphi(c)$ and $\psi(c)$	236
9.18. Statement of Result	236
Miscellaneous Exercises	237
Bibliography and Comments	240