CONTENTS

Part I

Chapter 1	Basic Structures	
1.1-1.12	Topology, groups, rings and vector spaces, 1-7	
1.13-1.14	Hamel basis, 7	
1.15-1.17	Linear mappings; projections, 7-9	
1.18	Inner product spaces, 10	
1.19-1.20	Normed spaces, metric spaces, Hilbert spaces, 11-14	
1.21	Completeness, 14	
1.22	Orthogonality, 20	
1.23	Continuous linear mappings, 26	
1.24	Linear functionals, and the Riesz-Fréchet Theorem 27	
1.25	Dual spaces, strong and weak convergence, $\sigma(X^*, X)$ topology, 28	
1.26-1.28	Adjoint operators, continuous linear operators, eigenvalues and eigenvectors, 30-36	
Chapter II	The General Problem of Approximation with Applications to the Class C of Continuous Functions	
2.1-2.3	Existence of elements of best approximation, 41-42	
2.4-2.5	Orthogonality to a subspace, Čebyšev polynomials, 44-45	
2.6	Uniqueness of best approximation, 49	
2.7-2.8	Convex hulls in \mathbb{R}^n , 51-53	

iv	Contents

	Haar spaces and unicity theorems, 54-55 Best approximation and extremal problems in other norms, uniform and strict convexity, 64
Chapter III	Characterization of an Element of Best Approximation
3.1	Best approximation in a normed linear space by elements of a linear subspace, 69
3.2	Characterization theorems, 70
3.3	Geometrical applications, 74
3.4	Topological linear spaces, 77
3.5	Convex sets and hyperplanes, 78
3.6-3.7	Cells, support and tangent hyperplanes, 79-83
3.8-3.9	The second theorem of characterization; extremal points, 83-86
3.10	Statement of the second theorem of characterization, 89
3.11	Geometrical interpretations; extremal hyperplanes, 91
3.12	Other characterization of elements of best approximation, 93
3.13	Orthogonality in general normed linear spaces, 95
3.14-3.16	Existence of elements of best approximation; proximinal linear
3.17	subspaces, 96-101 Uniqueness of elements of best approximation; Čebyšev and semi-Čebyšev subspaces, 103
3.18-3.20	k-dimensional $\mathcal{L}_G(x)$ sets, k semi-Čebyšev subspaces, 110-112
3.21-3.22	Čebyšev rank and k-strict convexity, 113
3.23-3.24	Interpolative best approximation, 117-118
3.25-3.30	The operators π_G ; the functionals e_G ; deviations and elements of ϵ -approximation, 119-137
Part II	
Chapter IV	Spline Functions
4.1-4.2	Spline functions, natural splines, 141-142
4.3	"First" integration formula, 143
4.4	Interpolation by splines, 144
4.5	Best approximation of linear functionals, Peano's theorem, 147
4.6	Best Quadrature formulae, 149
4.7	Exact Approximations, 151
4.8	Boundary conditions in interpolation problems, 155
4.9	Least square smoothing splines, 157
4.10	B-splines, 159
4.11	Interpolation from a computational point of view, 165
4.12	Multidimensional splines, 168

Chapter V	L-splines, Lg -splines and Error Analysis
5.1	Linear homogeneous systems, 173
5.2	Self-adjoint linear differential operators, 178
5.3-5.5	L-splines; the L-Hermite problem; integral relations, 181-188
5.6-5.8	Error bounds, 190-204
5.9	Lg-splines; Hermite-Birkhoff interpolation problems, 208
5.10	A basis for LL , 218
5.11	Generalized Taylor's formula, 224
5.12	Approximation of linear functionals, 230
Chapter VI	Cardinal Splines
6.1	Introduction and definition, 233
6.2	A fundamental function, generalized polynomials, 234
6.3	An eigenvalue problem, 237
6.4	The interpolation problem, 242
6.5	The fundamental splines, forward B-splines, exponential Euler
	splines, 243
6.6	Convergence results, 247
6.7	Cardinal Hermite interpolation, 250
6.8-6.10	Landau-type inequalities and their extremal functions; extensions, 258-267
6.11	Cardinal t-perfect L-splines, 269
Appendix	

Bibliography

Index