

# Contents

Preface	v
<b>1 Formulas from Algebra, Trigonometry and Analytic Geometry</b>	<b>1</b>
<b>H. Lennart Pearson</b>	
1.1 The Real Number System	1
1.2 The Complex Number System	2
1.3 Inequalities	5
1.4 Powers and Logarithms	5
1.5 Polynomial Equations	9
1.6 Rational Functions and Partial Fractions	15
1.7 Determinants and Solution of Systems of Linear Equations	16
1.8 Progressions	21
1.9 Binomial Theorem, Permutations and Combinations	22
1.10 The Trigonometric Functions	23
1.11 Analytic Geometry of Two-Space	40
1.12 Analytic Geometry of Three-Space	58
1.13 References and Bibliography	81
<b>2 Elements of Analysis</b>	<b>83</b>
<b>H. Lennart Pearson</b>	
2.1 Sequences	83
2.2 Infinite Series	85
2.3 Functions, Limits, Continuity	90
2.4 The Derivative	93
2.5 The Definite Integral	99
2.6 Methods of Integration	105
2.7 Improper Integrals	111
2.8 Partial Differentiation	115
2.9 Multiple Integrals	120
2.10 Infinite Products	126
2.11 Fourier Series	126
2.12 References and Bibliography	128

<b>3</b>	<b>Vector Analysis</b>	129
	<b>Gordon C. Oates</b>	
3.0	Introduction	129
3.1	Coordinate Systems	129
3.2	Vector Algebra	135
3.3	Vector Calculus	141
3.4	Successive Operations	156
3.5	Vector Fields	162
3.6	Summary	165
3.7	Bibliography	178
<b>4</b>	<b>Tensors</b>	179
	<b>Bernard Budiansky</b>	
4.0	Introduction	179
4.1	Vectors in Euclidean 3-D	179
4.2	Tensors in Euclidean 3-D	184
4.3	General Curvilinear Coordinates in Euclidean 3-D	191
4.4	Tensor Calculus	194
4.5	Theory of Surfaces	201
4.6	Classical Interlude	213
4.7	An Application: Continuum Mechanics	218
4.8	Tensors in $n$ -Space	223
4.9	Bibliography	225
<b>5</b>	<b>Functions of a Complex Variable</b>	226
	<b>A. Richard Seebass</b>	
5.0	Introduction	226
5.1	Preliminaries	226
5.2	Analytic Functions	229
5.3	Singularities and Expansions	242
5.4	Residues and Contour Integrals	248
5.5	Harmonic Functions and Conformal Mapping	257
5.6	Acknowledgments	268
5.7	References and Bibliography	269
<b>6</b>	<b>Ordinary Differential and Difference Equations</b>	271
	<b>Edward R. Benton</b>	
6.0	Introduction	271
6.1	Basic Concepts	271
6.2	First-Order Linear Differential Equations	279

6.3	Second Order Linear Differential Equations with Constant Coefficients	283
6.4	Second Order Linear Differential Equations with Variable Coefficients	290
6.5	Linear Equations of High Order and Systems of Equations	307
6.6	Eigenvalue Problems	315
6.7	Nonlinear Ordinary Differential Equations	323
6.8	Approximate Methods	327
6.9	Ordinary Difference Equations	336
6.10	References	342
<b>7</b>	<b>Special Functions</b>	<b>344</b>
	<b>Victor Barcion</b>	
7.0	Introduction	344
7.1	Exponential Integral and Related Functions	346
7.2	Gamma Function and Related Functions	348
7.3	Error Function and Related Functions	351
7.4	Bessel Functions	353
7.5	Modified Bessel Functions	359
7.6	Orthogonal Polynomials	361
7.7	Hypergeometric Functions and Legendre Functions	369
7.8	Elliptic Integrals and Functions	372
7.9	Other Special Functions	375
7.10	References and Bibliography	377
<b>8</b>	<b>First Order Partial Differential Equations</b>	<b>378</b>
	<b>Jirair Kevorkian</b>	
8.0	Introduction	378
8.1	Examples of First Order Partial Differential Equations	379
8.2	Geometrical Concepts, Qualitative Results	386
8.3	Quasilinear Equations	400
8.4	Nonlinear Equations	422
8.5	References	447
<b>9</b>	<b>Partial Differential Equations of Second and Higher Order</b>	<b>448</b>
	<b>Carl E. Pearson</b>	
9.0	Survey of Contents	448
9.1	Derivation Examples	448
9.2	The Second-Order Linear Equation in Two Independent Variables	453
9.3	More General Equations	459
9.4	Series Solutions	462

9.5	Transform Methods	465
9.6	The Perturbation Idea	467
9.7	Change of Variable	469
9.8	Green's Function	473
9.9	Potential Theory	482
9.10	Eigenvalue Problems	484
9.11	Characteristics	486
9.12	Variational Methods	493
9.13	Numerical Techniques	496
9.14	References	509
<b>10</b>	<b>Integral Equations</b>	<b>512</b>
	<b>Donald F. Winter</b>	
10.1	Introduction	512
10.2	Definitions and Classifications	513
10.3	Origin of Integral Equations	519
10.4	Nonsingular Linear Integral Equations	527
10.5	Singular Linear Integral Equations	547
10.6	Approximate Solution of Integral Equations	558
10.7	Nonlinear Integral Equations	564
10.8	References	569
<b>11</b>	<b>Transform Methods</b>	<b>571</b>
	<b>Gordon E. Latta</b>	
11.0	Introduction	571
11.1	Fourier's Integral Formula	571
11.2	Laplace Transforms	578
11.3	Linearity, Superposition, Representation Formulas	585
11.4	The Wiener-Hopf Technique	592
11.5	Abel's Integral Equation, Fractional Integrals, Weyl Transforms	601
11.6	Poisson's Formula, Summation of Series	606
11.7	Hilbert Transforms, Riemann-Hilbert Problem	609
11.8	Finite Transforms	622
11.9	Asymptotic Results	624
11.10	Operational Formulas	625
11.11	References	630
<b>12</b>	<b>Asymptotic Methods</b>	<b>631</b>
	<b>Frank W. J. Olver</b>	
12.1	Definitions	631
12.2	Integrals of a Real Variable	636
12.3	Contour Integrals	643

12.4	Further Methods for Integrals	648
12.5	Sums and Sequences	658
12.6	The Liouville-Green (or JWKB) Approximation	667
12.7	Differential Equations with Irregular Singularities	675
12.8	Differential Equations with a Parameter	680
12.9	Estimation of Remainder Terms	691
12.10	References and Bibliography	695
<b>13</b>	<b>Oscillations</b>	<b>697</b>
	<b>Richard E. Kronauer</b>	
13.0	Introduction	697
13.1	Lagrange Equations	697
13.2	Conservative Linear Systems, Direct Coupled	700
13.3	Systems with Gyroscopic Coupling	709
13.4	Mathieu-Hill Systems	712
13.5	Oscillations with Weak Nonlinearities	718
13.6	Oscillators Coupled by Weak Nonlinearity	732
13.7	References and Bibliography	745
<b>14</b>	<b>Perturbation Methods</b>	<b>747</b>
	<b>G. F. Carrier</b>	
14.1	Introduction	747
14.2	Perturbation Methods for Ordinary Differential Equations	748
14.3	Partial Differential Equations	756
14.4	Multiscaling Methods	764
14.5	Boundary Layers	780
14.6	Remarks	813
14.7	References	814
<b>15</b>	<b>Wave Propagation</b>	<b>815</b>
	<b>Wilbert Lick</b>	
15.0	Introduction	815
15.1	General Definitions and Classification of Waves	816
15.2	Physical Systems and Their Classification	821
15.3	Simple Waves: Nondispersive, Nondiffusive	829
15.4	Dispersive Waves	848
15.5	Diffusive Waves	862
15.6	References and Bibliography	875

<b>16</b>	<b>Matrices and Linear Algebra</b>	878
	<b>Tse-Sun Chow</b>	
16.1	Preliminary Considerations	878
16.2	Determinants	882
16.3	Vector Spaces and Linear Transformation	885
16.4	Matrices	889
16.5	Linear System of Equations	892
16.6	Eigenvalues and the Jordan Normal Form	897
16.7	Estimates and Determination of Eigenvalues	906
16.8	Norms	908
16.9	Hermitian Forms and Matrices	911
16.10	Matrices with Real Elements	913
16.11	Generalized Inverse	916
16.12	Commuting Matrices	917
16.13	Compound Matrices	920
16.14	Handling Large Sparse Matrices	921
16.15	References	925
<b>17</b>	<b>Functional Approximation</b>	928
	<b>Robin Esch</b>	
17.0	Introduction	928
17.1	Norms and Related Measures of Error	931
17.2	Relationship between Approximation on a Continuum and on a Discrete Point Set	935
17.3	Existence of Best Approximations	937
17.4	$L_2$ or Least-Mean-Square Approximation	938
17.5	Theory of Chebyshev Approximation	950
17.6	Chebyshev Approximation Methods Based on Characterization Properties	958
17.7	Use of Linear Programming in Chebyshev Approximation	967
17.8	$L_1$ Approximation	974
17.9	Piecewise Approximation without Continuity at the Joints	977
17.10	Approximation by Splines and Related Smooth Piecewise Functions	981
17.11	References	985
<b>18</b>	<b>Numerical Analysis</b>	988
	<b>A. C. R. Newbery</b>	
18.0	Introduction	988
18.1	General Information on Error Analysis	988
18.2	Linear Equation Systems	992
18.3	Eigenvalue and Lambda-Matrix Problems	999
18.4	Approximation and Interpolation	1003
18.5	Quadrature and Integral Equations	1015

18.6	Ordinary Differential Equations	1022
18.7	Nonlinear Functions of One Variable	1027
18.8	Nonlinear Equation Systems and Optimization	1032
18.9	Miscellaneous Topics	1037
18.10	References and Bibliography	1040
<b>19</b>	<b>Mathematical Models and Their Formulation</b>	<b>1044</b>
	<b>Frederic Y. M. Wan</b>	
19.1	Mathematical Modeling	1044
19.2	Groping in the Dark	1049
19.3	From the Simple to the Elaborate	1060
19.4	Try a Different Formulation	1081
19.5	Linearize with Care	1102
19.6	Stepping Beyond Reality	1116
19.7	Why Reinvent the Wheel?	1123
19.8	Better Robust Than Realistic	1135
19.9	References	1136
<b>20</b>	<b>Optimization Techniques</b>	<b>1140</b>
	<b>Juris Vagners</b>	
20.1	Introduction	1140
20.2	Parameter Optimization	1147
20.3	Dynamic Optimization, Necessary Conditions	1158
20.4	Extremal Fields, Sufficiency Conditions	1186
20.5	Computational Techniques	1196
20.6	Elements of Game Theory	1208
20.7	References	1213
<b>21</b>	<b>Probability and Statistics</b>	<b>1217</b>
	<b>L. Fisher</b>	
21.0	Introduction	1217
21.1	Probability Spaces	1217
21.2	Random Vectors and Random Variables	1220
21.3	Descriptive Statistics	1233
21.4	Statistical Inference	1233
21.5	The General Linear Model	1243
21.6	Some Other Techniques of Multivariate Analysis	1248
21.7	Parametric, Nonparametric, and Distribution-Free Statistical Tests	1253
21.8	Bayesian Statistics and Decision Theory	1262
21.9	Concluding Remarks	1269
21.10	References	1270
	<b>Index</b>	<b>1275</b>