

# Table of Contents

---

<b>Preface</b> .....	9
<b>Chapter 1 Computational arithmetic: the limitations and errors in operations with floating-point numbers</b> .....	13
1.1 Fixed-point arithmetic .....	14
1.2 Floating-point numbers .....	14
1.3 Rounding error of floating-point numbers .....	16
1.4 The frequency distribution of error .....	19
1.5 Transmitted error .....	22
1.6 Generated error .....	24
1.7 Error in arithmetic operations .....	29
1.8 Exceptional arithmetic conditions .....	35
1.9 Reprise .....	36
<b>Chapter 2 Function evaluation: summing series and working with continued fractions</b> .....	38
2.1 Truncation error .....	39
2.2 Infinite series .....	39
2.3 The control of error in series summation .....	42
2.3.1 Alternating series .....	44
2.3.2 Taylor series .....	46
2.3.3 Comparison series .....	47
2.3.4 An integral approximation .....	48
2.4 Generated error in series summation .....	53
2.5 Semi-convergent series .....	55
2.6 Choosing a series expansion .....	59
2.7 Continued fractions .....	63
2.8 Convergence of continued fractions .....	66
2.9 Generation of S-fractions .....	68
2.10 Recurrence relations and numerical instability .....	72
2.11 Reprise .....	76

<b>Chapter 3</b>	<b>Interpolation: curve drawing and function approximation. . . . .</b>	<b>78</b>
3.1	Curve fitting. . . . .	79
3.2	Polynomial interpolation formulae . . . . .	81
3.2.1	Techniques of polynomial evaluation . . . . .	87
3.2.2	Newton's form of the interpolating polynomial. . . . .	92
3.2.3	Tabular interpolation . . . . .	99
3.2.4	Osculatory or Hermitian interpolation . . . . .	102
3.3	Piecewise polynomial interpolation. . . . .	106
3.4	Interpolation by splines . . . . .	110
3.5	Interpolation and extrapolation by rational functions. . . . .	119
3.6	Parametric and periodic interpolation . . . . .	128
3.7	Interpolation and extrapolation of functions . . . . .	132
3.7.1	The error in Lagrangian interpolation . . . . .	133
3.7.2	Tchebyshev interpolation . . . . .	135
3.8	Minimax polynomials. . . . .	137
3.8.1	'Nearly minimax' polynomials. . . . .	138
3.8.2	Techniques of minimax approximation . . . . .	143
3.9	Minimax rational functions . . . . .	144
3.10	Other criteria of 'best' approximation. . . . .	146
3.11	Reprise . . . . .	147
<b>Chapter 4</b>	<b>Solving equations: finding the real zeros of a function of one variable . . . . .</b>	<b>149</b>
4.1	Bracketed methods of root finding . . . . .	150
4.1.1	The method of interval bisection . . . . .	151
4.1.2	The method of <i>regula falsi</i> . . . . .	153
4.1.3	Modified <i>regula falsi</i> . . . . .	155
4.2	Unbracketed methods of root finding . . . . .	161
4.2.1	The secant method . . . . .	162
4.2.2	Newton-Raphson's methods . . . . .	164
4.2.3	Global convergence of secant and Newton-Raphson's methods . . . . .	166
4.2.4	Fixed-point iteration . . . . .	169
4.2.5	Three-point iteration formulae. . . . .	171
4.2.6	Termination conditions . . . . .	174
4.3	The location of multiple roots . . . . .	175
4.4	Locating the real zeros of a polynomial. . . . .	178
4.4.1	Sturm's theorem. . . . .	179
4.4.2	A Sturm sequence of polynomials. . . . .	180
4.4.3	Treatment of multiple roots by Sturm sequences. . . . .	184
4.4.4	Polynomial deflation . . . . .	185
4.5	Optimization in one dimension . . . . .	187

4.5.1	Bracketing a local minimum . . . . .	188
4.5.2	Interval reduction by golden section . . . . .	189
4.6	Reprise . . . . .	191
<b>Chapter 5</b>	<b>Numerical quadrature: estimating the value of definite integrals</b> .	<b>193</b>
5.1	Quadrature based on equally-spaced integrand evaluations . . . . .	194
5.1.1	The Cotes formulae . . . . .	196
5.1.2	Composite Cotes formulae . . . . .	201
5.1.3	Romberg's method . . . . .	203
5.1.4	Integrals of periodic functions . . . . .	208
5.1.5	Gregory's formulae . . . . .	209
5.2	Quadrature involving unequally-spaced integrand evaluations. . . . .	213
5.2.1	Gaussian quadrature rules . . . . .	214
5.2.2	Formulae based on common-point sequences . . . . .	217
5.2.3	Transformation of the integrand. . . . .	220
5.3	Adaptive quadrature . . . . .	224
5.4	Areas and integrals defined by irregular discrete data . . . . .	229
5.5	Singular integrands and improper integrals . . . . .	237
5.6	Reprise . . . . .	239
<b>Bibliography</b>	. . . . .	<b>241</b>
<b>Appendix</b>	<b>Some numerical subroutines in BASIC</b> . . . . .	<b>249</b>
A.1	The subset of BASIC used. . . . .	249
A.2	How the subroutines are specified. . . . .	250
A.3	Subroutine specifications . . . . .	252
A.3.1	Machine arithmetic . . . . .	252
A.3.2	Evaluation techniques . . . . .	253
A.3.3	Conversion techniques . . . . .	257
A.3.4	Interpolation techniques. . . . .	259
A.3.5	Solving equations . . . . .	262
A.3.6	Numerical quadrature. . . . .	264
A.4	Subroutine listing . . . . .	265
<b>Problems</b>	. . . . .	<b>274</b>
Chapter 1.	. . . . .	274
Chapter 2.	. . . . .	283
Chapter 3.	. . . . .	301
Chapter 4.	. . . . .	322
Chapter 5.	. . . . .	335
<b>Hints and solutions</b>	. . . . .	<b>350</b>
Chapter 1.	. . . . .	350
Chapter 2.	. . . . .	356

Chapter 3. . . . .	373
Chapter 4. . . . .	394
Chapter 5. . . . .	404
<b>List of principle notations &amp; Index . . . . .</b>	<b>418</b>