TABLE OF CONTENTS

INTRODUCTION,

1 THE ELEMENTS OF THE THEORY OF DETERMINANTS, 3

- 1. Introduction, 3.
- 2. Fundamental definitions and notation, 3.
- 3. Minors and cofactors, 4.
- 4. The Laplace expansion of a determinant, 5.
- 5. Fundamental properties of determinants, 7.
- 6. The expansion of numerical determinants, 8.
- 7. The method of pivotal condensation, 10.
- 8. The solution of linear equations. Cramer's rule, 13.
- 9. Multiplication and differentiation of determinants, 15.

2 THE FUNDAMENTALS OF MATRIX ALGEBRA, 18

- 1. Introduction, 18.
- 2. Definition of a matrix, 18.
- 3. Principal types of matrices, 19.
- 4. Equality of matrices, addition and subtraction, 20.
- 5. Multiplication of matrices, 21.
- 6. Matrix division, the inverse matrix, 24.
- 7. Matrix division, 28.
- 8. The reversal law in transposed and reciprocal products, 29.
- 9. Diagonal matrices and their properties, 31.
- 10. Partitioned matrices and partitioned multiplication, 31.
- 11. Matrices of special types, 32.
- 12. The solution of n linear equations in n unknowns, 34.

3 MATRICES AND EIGENVALUE PROBLEMS, 38

- 1. Introduction, 38.
- 2. The eigenvalues of a square matrix, 38.
- 3. Geometrical interpretation of the eigenvalue problem, 40.
- 4. Invariants of the rotated matrix $[M]_r$, 45.
- 5. Algebraic discussion of the eigenvectors of a matrix, 46.
- 6. The eigenvalues of a real symmetric matrix are real numbers, 49.
- 7. Hermitian matrices, 50.
- 8. Orthogonal transformations, 51.
- 9. Symmetric matrices and quadratic forms, 53.
- 10. The transformation of a quadratic form into a sum of squares by means of an orthogonal transformation, 54.
- 11. Commutative matrices, 57.
- 12. Fundamental properties of the characteristic determinant and the characteristic equation of a matrix, 59.

4 THE CALCULUS OF MATRICES, 65

- 1. Introduction, 65.
- 2. Matrix polynomials, 65.
- 3. Infinite series of matrices, 66.
- 4. The convergence of series of matrices, 67.
- 5. The matric exponential function, 69.
- 6. The matric trigonometric and hyperbolic functions, 70.
- 7. The matric binomial theorem (commutative matrices), 72.
- 8. The Cayley-Hamilton theorem, 73.
- 9. Reduction of polynomials by the Cayley-Hamilton theorem, 75.
- 10. The inversion of a matrix by the use of the Cayley-Hamilton theorem, 77.
- 11. Functions of matrices and Sylvester's theorem, 78.
- 12. Applications of Sylvester's theorem, 82.
- 13. The use of the characteristic equation in evaluating functions of matrices, 83.
- 14. Differentiation and integration of matrices, 88.
- 15. Association of matrices with linear differential equations, 90.
- 16. The solution of difference equations by the use of matrices, 94.

5 MATRIX METHODS IN THE THEORY OF ELASTICITY, 100

- 1. Introduction, 100.
- 2. Stress. Definition and notation, 100.
- 3. Stresses in two dimensions, 102.
- 4. Stresses in three dimensions, 106.
- 5. Infinitesimal strain in three dimensions, 111.
- 6. The strain energy of the medium, 117.
- 7. The stress-strain relations for an elastic isotropic solid, 117.
- 8. The principal elastic constants, 120.
- 9. The strain expressed in terms of the stress, 122.
- 10. The elastic equations of motion, 122.
- 11. The elastic equations of equilibrium, 125.
- 12. The stresses on a semi-infinite solid strained by gravitational forces, 126.

6 MATRIX METHODS IN THE ANALYSIS OF STRUCTURES, 129

- 1. Introduction, 129.
- 2. The flexibility and stiffness matrices, 130.
- 3. Conservative bodies and the strain energy, 134.
- 4. Castigliano's theorem, 135.
- 5. The principal directions of loading at a point of an elastic body, 135.
- 6. The principle of minimum potential energy, 136.
- 7. Determination of the flexibility matrix of a complex structure, 137.
- 8. The force method of stress analysis, 138.
- 9. The modified flexibility matrix, 142.
- 10. Analysis of modified structures, 144.
- 11. Modifications of the elements of the structure, 147.
- 12. The flexibility of the modified structure, 150.
- 13. The displacement method of stress analysis, 151.
- 14. The effect of the application of initial stresses p_0 with the constraint S = 0, 154.
- 15. Analysis of modifications of the structure, 155.
- 16. The stiffness matrix of the modified structure, 157.
- 17. Special types of modification of the structure, 157.

APPLICATIONS OF MATRICES TO CLASSICAL MECHANICS, 163

- 1. Introduction, 163.
- 2. The matrix representation of a vector product, 163.
- 3. Rotation of rectangular axes, 165.
- 4. Kinematics and angular velocity, 166.
- 5. Change of reference axes in two dimensions, 170.
- 6. Successive rotations, 172.
- Angular coordinates of a three-dimensional reference frame, 172.
- 8. The transformation matrix [L] and instantaneous angular velocities expressed in angular coordinates, 174.
- 9. The components of velocity and acceleration, 175.
- 10. The kinetic energy of a rigid body, 177.
- 11. Principal axes and the moments of inertia, 179.
- 12. Transformation of forces and couples, 181.
- 13. The equations of motion of a rigid body, 183.
- 14. Transformation of the equation of motion to moving axes, 186.
- 15. General form of Euler's equations, 189.
- 16. Motion relative to the surface of the earth, 193.
- 17. The plumb line, 196.
- 18. Motion of a free particle near the surface of the rotating earth, 198.
- 19. The Foucault pendulum, 201.
- 20. The motion of a top, 203.

8 APPLICATIONS OF MATRICES TO VIBRATION PROBLEMS, 210

- 1. Introduction, 210.
- 2. Transformation of coordinates, 210.
- 3. Lagrange's equations of motion, 212.
- 4. Electrical and mechanical analogies, 214.
- 5. Systems having two degrees of freedom (conservative case), 218.
- 6. The general case: mass-weighted coordinates, 224.
- The vibration of conservative systems with dynamic coupling, 227.

APPLICATIONS OF MATRICES TO VIBRATION PROBLEMS (Cont'd)

- 8. Fundamental properties of the modal matrix [A], 231.
- 9. The case of three coupled pendulums, 234.
- 10. The case of zero frequency, 238.
- 11. The use of functions of matrices in the theory of vibrations, 241.
- 12. Use of functions of matrices in the case of dynamic coupling, 247.
- 13. The case of multiple roots of the characteristic equation, 249.
- 14. The oscillations of a symmetric electric circuit, 251.
- 15. Nonconservative systems. Vibrations with viscous damping, 254.
- 16. The motion of a general damped linear dynamic system, 256.
- 17. The use of matrix iteration to determine the frequencies and modes of oscillation of linear conservative systems, 262.
- 18. Numerical example, 264.
- 19. Determination of the higher modes: the sweeping matrix, 267.
- 20. A numerical example of the iteration procedure, 270.
- 21. The analysis of a class of symmetric damped linear systems, 279.
- 22. The Routh-Hurwitz stability criterion, 282.
- 23. The location of the eigenvalues of a matrix, 285.

9 THE STEADY-STATE SOLUTION OF THE GENERAL n-MESH CIRCUIT, 288

- 1. Introduction, 288.
- 2. The general network, 288.
- 3. Formulation of the problem in terms of mesh currents, 290
- 4. The canonical equations and their solution, 292.
- 5. Steady-state solution, one applied sinusoidal electromotive force, 295.
- 6. Case of *n* sinusoidal electromotive forces of different phases, 298.
- 7. Voltages of different frequencies simultaneously impressed, 298.

10 THE TRANSIENT SOLUTION OF THE GENERAL NETWORK, 305

- 1. Introduction, 305.
- 2. The heuristic process, 305.
- 3. The determinantal equation, 306.
- 4. The normal modes, 306.
- 5. The modal matrix, 307.
- 6. The relation between the amplitudes, 307.
- 7. Determination of the arbitrary constants, 308.
- 8. The case of repeated roots of the determinantal equation, 311.
- 9. The energy functions of the general network, 313.

11 THE DISSIPATIONLESS NETWORK, 316

- 1. Introduction, 316.
- 2. The canonical equations and the steady-state solution, 316.
- 3. The transient solution, 319.
- 4. Determination of the fundamental frequency, 322.
- 5. Completion of the solution, 323.
- 6. Evaluation of the arbitrary constants, 325.
- 7. Normal coordinates, 326.
- 8. Illustrative example, 327.
- 9. Effect of small resistance terms, 333.
- 10. The potential function, 334.
- 11. Computation of the attenuation constants, 336.

12 THE STEADY-STATE ANALYSIS OF FOUR-TERMINAL NETWORKS, 337

- 1. Introduction, 337.
- 2. The general equations, 337.
- 3. Alternative form of the equations, 340.
- 4. Interconnection of four-terminal networks, 341.
- 5. The chain matrices of common structures, 347.
- 6. The homographic transformation, 351. 7. Cascade connection of dissymmetrical networks, 352.
- 8. Attenuation and pass bands, 359.
- 9. The smooth transmission lines, 361.
- 10. The general ladder network, 363.

13 STEADY-STATE SOLUTION OF MULTICONDUCTOR LINES, 365

- 1. Introduction, 365.
- 2. The coefficients of capacity and induction, 366.
- 3. The electromagnetic coefficients, 367.
- 4. The general differential equations, 368.
- 5. The steady-state equations, 370.
- 6. Solution of the equations, boundary conditions, 371.
- 7. The determinantal equation, 375.
- 8. Transformation of the basic equations, 378.
- 9. Solution in terms of terminal impedances, 381.
- 10. General considerations, 383.

14 TRANSIENT ANALYSIS OF MULTICONDUCTOR LINES, 384

- 1. Introduction, 384.
- 2. The general equations, 384.
- 3. The dissipationless case, 386.
- 4. The general case, 390.
- 5. The case of ring symmetry, 392.
- 6. General boundary conditions, 395.
- 7. General considerations, 397.

APPENDIX

THE ELEMENTS OF THE
THEORY OF LAPLACE TRANSFORMS, 401

APPENDIX

2 THE BASIC THEOREMS OF THE LAPLACE TRANSFORMS, 404

APPENDIX

3 TABLE OF BASIC TRANSFORMS, 407