

Contents

1	Introduction	
1.1	Introduction	1
1.2	A geometrical example	1
1.3	Small vibrations	3
1.4	An example in information system design	5
1.5	An eigenproblem in non-linear optimization	6
1.6	An example from mathematical economics	7
1.7	A Sturm-Liouville problem	8
2	Background theory	
2.1	Introduction	11
2.2	Eigenvalues and eigenvectors	13
2.3	Similarity transformations	15
2.4	The Jordan canonical form	17
2.5	Some properties of Hermitian matrices	19
2.6	Vector and matrix norms	20
2.7	Theorems on bounds for the eigenvalues	22
2.8	Condition of the eigenvalue problem	23
2.9	Stability of similarity transformation methods	24
3	Reductions and transformations	
3.1	Introduction	26
3.2	Elementary operation matrices	26
3.3	Elementary unitary matrices	28
3.4	Elementary Hermitian matrices	29
3.5	Gaussian elimination	30
3.6	Unitary decomposition of a matrix	31
3.7	Elementary similarity transformations	35
4	Methods for the dominant eigenvalue	
4.1	Introduction	38
4.2	The power method	38
4.3	Shift of origin	42
4.4	Aitken's acceleration device	43
4.5	The Rayleigh quotient	45

5 Methods for the subdominant eigenvalue	
5.1 Introduction	47
5.2 Deflation	47
5.3 Simultaneous iteration for real symmetric matrices	51
6 Inverse iteration	
6.1 Introduction	56
6.2 Inverse iteration for an eigenvalue	56
6.3 Computational procedure for inverse iteration	58
7 Jacobi's method	
7.1 Introduction	63
7.2 Jacobi's algorithm	64
7.3 Variants of the Jacobi algorithm	67
7.4 The maximizing property of the classical Jacobi algorithm	68
7.5 Calculation of the eigenvectors	69
8 Givens' and Householder's methods	
8.1 Introduction	71
8.2 Givens' method	71
8.3 Householder's method	74
8.4 Reduction of a Hermitian matrix	77
9 Eigensystem of a symmetric tridiagonal matrix	
9.1 Introduction	79
9.2 Sturm sequences and bisection	80
9.3 Eigenvectors of a tridiagonal matrix	84
10 The LR and QR algorithms	
10.1 Introduction	85
10.2 The LR algorithm	85
10.3 The QR algorithm	87
10.4 The QR algorithm with shifts	89
10.5 Analysis of convergence	90
11 Extensions of Jacobi's method	
11.1 Introduction	97
11.2 Normal matrices	97
11.3 General matrices	99

12 Extensions of Givens' and Householder's methods	
12.1 Introduction	102
12.2 Reduction to upper Hessenberg form	102
12.3 Further reduction to tridiagonal form	105
12.4 Evaluation of the characteristic polynomial	106
12.5 Computation of the eigenvalues	109
12.6 Evaluation of eigenvectors	113
13 QR algorithm for Hessenberg matrices	
13.1 Introduction	114
13.2 QR algorithm for a complex Hessenberg matrix	114
13.3 Double QR algorithm for a real Hessenberg matrix	115
14 Generalized eigenvalue problems	
14.1 Introduction	120
14.2 Parameterized matrices	122
14.3 The eigenvalue problem $A\mathbf{x} = \lambda B\mathbf{x}$	122
14.4 The eigenvalue problem $AB\mathbf{x} = \lambda\mathbf{x}$	126
15 Available implementations	
15.1 Introduction	128
15.2 Library routines	128
References	130
Index	131