

## Contents

Preface . . . . .	9
1. The Concept of Measurement . . . . .	15
1.1 Properties and Manifestations . . . . .	15
1.2 Representation of Properties by Language . . . . .	16
1.3 Relational Systems . . . . .	18
1.4 Indifference, Equivalence and Congruence Relations . . . . .	20
1.5 Homomorphisms and Isomorphisms . . . . .	22
1.6 Definition of Scales . . . . .	26
1.7 Uniqueness of Scales . . . . .	27
1.8 Some Types of Scales . . . . .	28
1.9 Multidimensional Measurement . . . . .	30
1.10 Fundamental and Derived Measurement . . . . .	31
1.11 Practical Performance of Measurement . . . . .	32
2. Meaningfulness . . . . .	34
2.1 Meaningful Relations . . . . .	34
2.2 Invariant Relations . . . . .	35
2.3 Examples . . . . .	40
2.4 Meaningful Statistics . . . . .	41
2.5 Examples of Meaningful and Meaningless Statistics . . . . .	45
2.6 Meaningfully Parametrized Relations . . . . .	50
2.7 Examples on Meaningful Parametrization . . . . .	52
3. Topology on Ordered Sets . . . . .	57
3.1 Topological Spaces . . . . .	57
3.2 Order-Relations . . . . .	58
3.3 Interval Topology . . . . .	60
3.4 Relative Topology and Interval Topology . . . . .	62
3.5 Order-Completeness and Connectedness . . . . .	63
3.6 Continuous and Monotone Maps . . . . .	65
3.7 Product Spaces . . . . .	68
3.8 Separability and Countable Base . . . . .	70
4. Nominal and Ordinal Scales . . . . .	74
4.1 Nominal Scales . . . . .	74
4.2 Ordinal Scales . . . . .	74
4.3 Ordinal Scales in Practice . . . . .	76
4.4 Continuity of Ordinal Scales . . . . .	77

<b>5. Operations . . . . .</b>	<b>80</b>
5.1 Monotony, Continuity and Separability . . . . .	80
5.2 Algebraic Properties of Binary Operations . . . . .	82
5.3 Mappings and Algebraic Properties . . . . .	87
5.4 Distributivity and Bisymmetry . . . . .	91
5.5 Additive Operations . . . . .	94
<b>6. The Theory of Interval Scales Based on Operations . . . . .</b>	<b>97</b>
6.1 Existence and Uniqueness of Interval Scales . . . . .	97
6.2 Inherent Zero-Points . . . . .	99
6.3 Zero-Points from Endomorphisms . . . . .	100
6.4 Simultaneous Arithmetic and Geometric Middling . . . . .	102
6.5 Joined Scales . . . . .	104
6.6 The Empirical Status of Axioms . . . . .	106
<b>7. Psychophysical Applications of Interval Scales Based on Operations . . . . .</b>	<b>120</b>
7.1 Additive Operations . . . . .	120
7.2 Middling Operations . . . . .	120
7.3 Endomorphisms . . . . .	124
7.4 Empirical Comparison of Arithmetic and Geometric Bisection Scales . . . . .	126
7.5 Cross-Modality and the Psychophysical Law . . . . .	127
<b>8. Order Relations in Product Sets . . . . .</b>	<b>129</b>
8.1 Order Relations in $A$ Derived from Order Relations in $A \times A$ . . . . .	129
8.2 $n$ -tuple-Conditions . . . . .	132
8.3 Continuity Conditions . . . . .	133
8.4 Relations between Derived and Given Order . . . . .	134
8.5 Order Relations in $A_1 \times A_2$ . . . . .	136
8.6 Order Relations in $A_1 \times \dots \times A_k$ . . . . .	140
<b>9. The Theory of Interval Scales Based on Distances . . . . .</b>	<b>143</b>
9.1 Distance Systems . . . . .	143
9.2 Distance Systems and Operations . . . . .	144
9.3 Existence and Uniqueness of Interval Scales Based on Distances . . . . .	147
9.4 Conjoint Measurement . . . . .	148
9.5 Empirical Status of Axioms for Product Structures . . . . .	152
<b>10. Canonical Representations . . . . .</b>	<b>155</b>
10.1 The Principle of Canonical Representations . . . . .	155
10.2 Uniqueness of Translation Invariant Representations . . . . .	152
10.3 Existence of Translation Invariant Representations . . . . .	158
10.4 Linear Representations . . . . .	161
<b>11. Scales Derived from Response . . . . .</b>	<b>165</b>
11.1 Response, Judgment and Valuation . . . . .	165
11.2 A Discriminal Dispersion Model . . . . .	167

11.3	Measurement of Stimuli . . . . .	170
11.4	Simultaneous Measurement of Stimuli and Subjects . . . . .	171
11.5	Matching of Stimuli . . . . .	172
11.6	Paired Comparisons . . . . .	173
11.7	Logit Models . . . . .	175
11.8	The Method of just Noticeable Differences . . . . .	177
11.9	Choices . . . . .	180
11.10	Rankings . . . . .	184
11.11	Similarity Response . . . . .	189
11.12	Dichotomous Valuation . . . . .	190
12.	Events, Utility and Subjective Probability . . . . .	195
12.1	The Algebra of Events . . . . .	195
12.2	The Space of Wagers . . . . .	201
12.3	Compound Wagers . . . . .	205
12.4	Preliminary Lemmas . . . . .	208
12.5	Theorems on Utility and Subjective Probability . . . . .	210
12.6	The MORGENTHORN-VON NEUMANN Approach . . . . .	213
12.7	Concluding Remarks . . . . .	218