

Contents

List of Symbols	xi
List of Figures	xiii
1 Introduction	1
1.1 Why Use Templates?	2
1.2 What Methods Are Covered?	3
2 Iterative Methods	5
2.1 Overview of the Methods	5
2.2 Stationary Iterative Methods	7
2.2.1 The Jacobi Method	8
2.2.2 The Gauss-Seidel Method	9
2.2.3 The Successive Overrelaxation Method	10
2.2.4 The Symmetric Successive Overrelaxation Method	12
2.2.5 Notes and References	12
2.3 Nonstationary Iterative Methods	14
2.3.1 Conjugate Gradient Method (CG)	14
2.3.2 MINRES and SYMMLQ	16
2.3.3 CG on the Normal Equations, CGNE and CGNR	18
2.3.4 Generalized Minimal Residual (GMRES)	18
2.3.5 BiConjugate Gradient (BiCG)	21
2.3.6 Quasi-Minimal Residual (QMR)	23
2.3.7 Conjugate Gradient Squared Method (CGS)	25
2.3.8 BiConjugate Gradient Stabilized (Bi-CGSTAB)	27
2.3.9 Chebyshev Iteration	28
2.4 Summary of the Methods	29
2.5 A short history of Krylov methods	34
2.6 Survey of recent Krylov methods	35

3	Preconditioners	39
3.1	The why and how	39
3.1.1	Cost trade-off	39
3.1.2	Theoretical prerequisites on preconditioners	40
3.2	Jacobi Preconditioning	40
3.2.1	Block Jacobi Methods	41
3.2.2	Discussion	41
3.3	SSOR preconditioning	41
3.4	Incomplete Factorization Preconditioners	42
3.4.1	Creating an incomplete factorization	42
3.4.2	Point incomplete factorizations	43
3.4.3	Block factorization methods	45
3.4.4	Incomplete LQ factorizations	48
3.5	Polynomial preconditioners	48
3.6	Other preconditioners	49
3.6.1	Preconditioning by the symmetric part	49
3.6.2	The use of fast solvers	49
3.6.3	Alternating Direction Implicit methods	50
4	Related Issues	51
4.1	Complex Systems	51
4.2	Stopping Criteria	51
4.2.1	More Details about Stopping Criteria	52
4.2.2	When $r^{(i)}$ or $\ r^{(i)}\ $ is not readily available	55
4.2.3	Estimating $\ A^{-1}\ $	56
4.2.4	Stopping when progress is no longer being made	56
4.2.5	Accounting for floating point errors	57
4.3	Data Structures	57
4.3.1	Survey of Sparse Matrix Storage Formats	58
4.3.2	Matrix vector products	62
4.3.3	Sparse Incomplete Factorizations	65
4.4	Parallelism	68
4.4.1	Inner products	68
4.4.2	Vector updates	70
4.4.3	Matrix-vector products	70
4.4.4	Preconditioning	70
4.4.5	Wavefronts in the Gauss-Seidel and Conjugate Gradient methods	71
4.4.6	Blocked operations in the GMRES method	72

5	Remaining topics	73
5.1	The Lanczos Connection	73
5.2	Block Iterative Methods	74
5.3	Reduced System Preconditioning	75
5.4	Domain Decomposition Methods	76
5.4.1	Overlapping Subdomain Methods	76
5.4.2	Non-overlapping Subdomain Methods	78
5.4.3	Multiplicative Schwarz Methods	80
5.5	Multigrid Methods	81
5.6	Row Projection Methods	82
A	Obtaining the Software	83
B	Overview of the BLAS	85
C	Glossary	87
C.1	Notation	92
	Bibliography	95
	Index	110