

Contents

Foreword	xi
Preface	xiii
Notation	xv
1 General Presentation	1
1.1 Coupling	1
1.2 Chaotic computations	2
1.3 Computability in finite precision	2
1.4 Numerical quality of computations	2
1.5 Role of singularities	3
1.6 Spectral instability and nonnormality	3
1.7 Influence on numerical software	4
1.8 Qualitative computing	4
1.9 Experimental mathematics	5
1.10 Sense of errors: For a rehabilitation of finite precision computations	5
2 Computability in Finite Precision	7
2.1 Well-posed problems	7
2.2 Approximations	8
2.3 Convergence in exact arithmetic	8
2.4 Computability in finite precision	11
2.5 Gaussian elimination	15
2.6 Forward error analysis	17
2.7 The influence of singularities	24
2.8 Numerical stability in exact arithmetic	27
2.9 Computability in finite precision for iterative and approximate methods	28
2.10 The limit of numerical stability in finite precision	30
2.11 Arithmetically robust convergence	31
2.12 The computed logistic	33
2.13 Bibliographical comments	37

3	Measures of Stability for Regular Problems	39
3.1	Choice of data and class of perturbations	40
3.2	Choice of norms: Scaling	40
3.3	Conditioning of regular problems	43
3.4	Simple roots of polynomials	45
3.5	Factorizations of a complex matrix	48
3.6	Solving linear systems	49
3.7	Functions of a square matrix	52
3.8	Concluding remarks	55
3.9	Bibliographical comments	56
4	Computation in the Neighbourhood of a Singularity	57
4.1	Singular problems that are well posed	57
4.2	Condition numbers of Hölder singularities	58
4.3	Computability of ill-posed problems	60
4.4	Singularities of $z \mapsto A - zI$	60
4.5	Distances to singularity	65
4.6	Unfolding of singularity	69
4.7	Spectral portraits	69
4.8	Bibliographical comments	70
5	Arithmetic Quality of Reliable Algorithms	71
5.1	Forward and backward analyses	71
5.2	Backward error	71
5.3	Quality of reliable software	72
5.4	Formulae for backward errors	74
5.5	Influence of the class of perturbations	78
5.6	Iterative refinement for backward stability	82
5.7	Robust reliability and arithmetic quality	84
5.8	Bibliographical comments	85
6	Numerical Stability in Finite Precision	87
6.1	Iterative and approximate methods	87
6.2	Numerical convergence of iterative solvers	87
6.3	Stopping criteria in finite precision	89
6.4	Robust convergence	91
6.5	The computed logistic revisited	94
6.6	Care of use	95
6.7	Bibliographical comments	96
7	Software Tools for Round-off Error Analysis in Algorithms	97
7.1	A historical perspective	97
7.2	Assessment of the quality of numerical software	98
7.3	Backward error analysis in libraries	99
7.4	Sensitivity analysis	99
7.5	Interval analysis	100

7.6	Probabilistic models	100
7.7	Computer algebra	102
7.8	Bibliographical comments	102
8	The Toolbox PRECISE for Computer Experimentation	103
8.1	What is PRECISE?	104
8.2	Module for backward error analysis	105
8.3	Sample size	114
8.4	Backward analysis with PRECISE	115
8.5	Dangerous border and unfolding of a singularity	120
8.6	Summary of module 1	123
8.7	Bibliographical comments	124
9	Experiments with PRECISE	125
9.1	Format of the examples	125
9.2	Backward error analysis for linear systems	126
9.3	Computer unfolding of singularity	136
9.4	Dangerous border and distance to singularity	140
9.5	Roots of polynomials	144
9.6	Eigenvalue problems	150
9.7	Conclusion	157
9.8	Bibliographical comments	157
10	Robustness to Nonnormality	159
10.1	Nonnormality and spectral instability	159
10.2	Nonnormality in physics and technology	164
10.3	Convergence of numerical methods in exact arithmetic	166
10.4	Influence on numerical software	167
10.5	Bibliographical comments	173
11	Qualitative Computing	175
11.1	Sensitivity of pseudosolutions for $F(x) = y$	176
11.2	Pseudospectra of matrices	177
11.3	Pseudozeroes of polynomials	182
11.4	Divergence portrait for the complex logistic iteration	188
11.5	Qualitative assessment of a Jordan form	189
11.6	Beyond linear perturbation theory	194
11.7	Bibliographical comments	197
12	More Numerical Illustrations with PRECISE	199
	Annex: The Toolbox PRECISE with PRECISE	211
	Bibliography	219
	Index	233