

Contents

1. Introduction, 1

1.1	Purpose, Scope, and Structure of Text	1
1.2	Notation	1
1.3	Classification of Differential Equations	3
1.4	Boundary and Initial Conditions	5

2. Basic Concepts, 9

2.1	Introduction	9
2.2	Polynomial Approximations	9
2.2.1	One-dimensional Theory	9
2.2.2	Two-dimensional Polynomial Approximation	17
2.3	The Finite Difference Approximation	22
2.3.1	One-dimensional Differences	23
2.3.2	Two-dimensional Finite Differences	26
2.4	The Method of Weighted Residuals	30
2.4.1	Introduction	30
2.4.1.1	Galerkin's Method	31
2.4.1.2	The Collocation Method	31
2.4.2	Basis Functions	32
2.4.2.1	General	32
2.4.2.2	Piecewise Linear Basis Functions	33
2.4.2.3	Higher-Order Basis Functions	36
2.5	A Boundary Value Problem	38
2.5.1	The Finite Difference Approach	38
2.5.2	The Galerkin Approximation	42
2.5.3	The Collocation Approximation	49
2.5.4	Comparing the Three Methods	51
2.6	Consistency, Stability, and Convergence	52
2.6.1	The Finite Difference Approximation	53
2.6.2	The Galerkin Approximation	54

2.6.3	The Collocation Approximation	60
2.7	Summary	63
3. The Finite Element Concept in Two Dimensions, 67		
3.1	Introduction	67
3.2	Rectangular Finite Elements	67
3.2.1	Product-Type Basis Functions	67
3.2.2	Serendipity Basis Functions	70
3.3	Triangular Elements	71
3.3.1	Lagrangian-Type Basis Functions	71
3.3.2	Hermitian-Type Basis Functions	74
3.3.3	Error Estimates	75
3.4	Isoparametric Transformations	77
3.5	Numerical Integration	82
4. Elliptic Partial Differential Equations, 85		
4.1	Introduction	85
4.2	The Finite Difference Approximation	85
4.2.1	General Formulation	85
4.2.2	Matrix Form of the Finite Difference Equations	88
4.2.3	A Numerical Example	91
4.3	The Galerkin Approximation	93
4.3.1	General	93
4.3.2	Implementation of the Method	94
4.4	The Collocation Approximation	97
4.4.1	General	97
4.4.2	Implementation of the Collocation Method	99
4.5	Solution of Linear Systems of Equations	100
4.5.1	General	100
4.5.2	Direct Methods	101
4.5.3	Iterative Methods	102
4.5.4	The Alternating Direction Implicit Method	104
4.6	Boundary Elements	106
4.6.1	The Boundary Integral	106
4.6.2	Boundary Element Formulation	110
4.6.3	Integration Methodology	111
4.6.4	Example Problem	112

4.6.5	Nonhomogeneous Operators	116
4.6.6	Summary	117
4.7	Comparison of the Methods	117

5. Parabolic Partial Differential Equations, 123

5.1	Introduction	123
5.2	The Finite Difference Approximation	124
5.2.1	Derivation of the Finite Difference Approximation	124
5.2.2	Stability of Finite Difference Approximations	128
5.2.3	Two-dimensional Problems	131
5.2.4	A Numerical Example	134
5.3	The Galerkin Approximation	136
5.3.1	General	136
5.3.2	Numerical Implementation	138
5.4	The Collocation Method	140
5.4.1	General	140
5.4.2	A Numerical Example	141
5.5	The Optimal θ Method	144
5.6	Conclusion	147

6. Hyperbolic Partial Differential Equations, 153

6.1	Introduction	153
6.2	The Method of Characteristics	154
6.3	Finite Difference Approximations	158
6.3.1	Equations of the First Order	158
6.3.2	One-dimensional Gas Dynamics	162
6.3.3	Second-Order Equations	164
6.4	The Galerkin Approximation	168
6.4.1	General	168
6.4.2	First-Order Equations	169
6.4.3	Second-Order Equations	171
6.5	The Collocation Method	172
6.6	Conclusion	174

7. Nonlinear and Singular Equations, 179

7.1	Introduction	179
7.2	Nonlinear Equations	179

7.2.1	General	179
7.2.2	Picard Iteration	181
7.2.3	The Newton Method	183
7.2.4	An Alternating Direction Implicit Method	184
7.3	Singularities	187
7.3.1	General	187
7.3.2	Reentrant Corners	188
7.3.3	Singular Coefficients	189

Index, 199