

Contents

Preface	v
Chapter I: Variational Formulations and Finite Element Methods	
§1. Classical Methods	1
§2. Model Problems and Elementary Properties of Some Functional Spaces	4
§3. Duality Methods	11
3.1. Generalities	11
3.2. Examples for symmetric problems	13
3.3. Duality methods for nonsymmetric bilinear forms	23
§4. Domain Decomposition Methods, Hybrid Methods	24
§5. Augmented Variational Formulations	30
§6. Transposition Methods	33
§7. Bibliographical remarks	35
Chapter II: Approximation of Saddle Point Problems	
§1. Existence and Uniqueness of Solutions	36
1.1. Quadratic problems under linear constraints	37
1.2. Extensions of existence and uniqueness results	44
§2. Approximation of the Problem	51
2.1. Basic results	51
2.2. Error estimates for the basic problem	54
2.3. The inf-sup condition: criteria	57
2.4. Extensions of error estimates	61
2.5. Various generalizations of error estimates	63

2.6. Perturbations of the problem, nonconforming methods.....	65
2.7. Dual error estimates.....	70
§3. Numerical Properties of the Discrete Problem	73
3.1. The matrix form of the discrete problem	73
3.2. Eigenvalue problem associated with the inf-sup condition ..	75
3.3. Is the inf-sup condition so important?	78
§4. Solution by Penalty Methods, Convergence of Regularized Problems.....	80
§5. Iterative Solution Methods. Uzawa's Algorithm	87
5.1. Standard Uzawa's algorithm	87
5.2. Augmented Lagrangian algorithm	87
§6. Concluding Remarks	88
 Chapter III: Function Spaces and Finite Element Approximations	89
§1. Properties of the spaces $H^s(\Omega)$ and $H(\text{div}; \Omega)$	89
1.1. Basic results	89
1.2. Properties relative to a partition of Ω	93
1.3. Properties relative to a change of variables	95
§2. Finite Element Approximations of $H^1(\Omega)$ and $H^2(\Omega)$	99
2.1. Conforming methods	99
2.2. Nonconforming methods.....	107
2.3. Nonpolynomial approximations: Spaces $\mathcal{L}_k^s(\mathfrak{E}_h)$	111
2.4. Scaling arguments	111
§3. Approximations of $H(\text{div}; \Omega)$	113
3.1. Simplicial approximations of $H(\text{div}; K)$	113
3.2. Rectangular approximations of $H(\text{div}; K)$	119
3.3. Interpolation operator and error estimates	124
3.4. Approximation spaces for $H(\text{div}; \Omega)$	131
§4. Concluding Remarks	132
 Chapter IV: Various Examples	133
§1. Nonstandard Methods for Dirichlet's Problem	134
1.1. Description of the problem.....	134
1.2. Mixed finite element methods for Dirichlet's problem	135
1.3. Primal hybrid methods	140
1.4. Dual hybrid methods	149
§2. Stokes Problem	155
§3. Elasticity Problems	159
§4. A Mixed Fourth-Order Problem	164
4.1. The $\psi-\omega$ biharmonic problem	164
§5. Dual Hybrid Methods for Plate Bending Problems	167
 Chapter V: Complements on Mixed Methods for Elliptic Problems	177
§1. Numerical Solutions	177
1.1. Preliminaries	177
1.2. Interelement multipliers.....	178

§2. A Brief Analysis of the Computational Effort	182
§3. Error Analysis for the Multiplier	186
§4. Error Estimates in Other Norms	191
§5. Application to an Equation Arising from Semiconductor Theory	193
§6. How Things Can Go Wrong	195
§7. Augmented Formulations	198
Chapter VI: Incompressible Materials and Flow Problems	200
§1. Introduction	201
§2. The Stokes Problem as a Mixed Problem	202
2.1. Mixed Formulation	202
§3. Examples of Elements for Incompressible Materials	206
3.1. Simple examples	207
§4. Standard Techniques of Proof for the inf-sup Condition	219
4.1. General results	224
4.2. Higher order methods	227
§5. Macroelement Techniques and Spurious Pressure Modes	228
5.1. Some remarks about spurious pressure modes	228
5.2. An abstract convergence result	231
5.3. Macroelement techniques	235
5.4. The bilinear velocity-constant pressure (Q_1-P_0) element	240
5.5. Other stabilization procedures, (Augmented Formulations)	246
§6. An Alternative Technique of Proof and Generalized Taylor-Hood Element	252
§7. Nearly Incompressible Elasticity, Reduced Integration Methods and Relation with Penalty Methods	258
7.1. Variational formulations and admissible discretizations	258
7.2. Reduced integration methods	259
7.3. Effects of inexact integration	263
§8. Divergence-Free Basis, Discrete Stream Functions	267
§9. Other Mixed and Hybrid Methods for Incompressible Flows	272
Chapter VII: Other Applications	274
§1. Mixed Methods for Linear Thin Plates	274
§2. Mixed Methods for Linear Elasticity Problems	282
§3. Moderately Thick Plates	295
3.1. Generalities	295
3.2. Discretization of the problem	303
3.3. Continuous Pressure Approximations	322
3.4. Discontinuous Pressure Approximations	322
References	324
Index	344