Contents

1. The differential equations

1.1 Introduction 1
1.2 Equations which are first order in time 1
1.3 The dynamic vibration equation 5
1.4 Eigenvalue bounds 10
Exercises 13
References 14

2. p-Step and p-stage schemes

2.1 Introduction 16
2.2 Consistency 20
2.3 Stability 22
2.4 Effect of mass lumping 25
2.5 Effect of artificial damping 28
2.6 Stability of the θ-method 30
2.7 The Newmark algorithm 35
2.8 Stability—general considerations 37
2.9 Lax Equivalence Theorem 40
Exercises 45
References 46

3. Newmark and the θ-method

3.1 Introduction 49
3.2 The Newmark method 51
3.3 The θ-method applied to the dynamic vibration equation 56
3.4 Accuracy and stability 59
Summary 66
Exercises 66
References 66

4. General theory and higher order p-step and p-stage methods

4.1 Introduction 68
4.2 The dynamic vibration equation—general formulae for amplitude and phase errors 70

4.3 Newmark and the θ-method (continued) 76
4.4 Newmark variations 82
4.5 Generalized Newmark GNpj 94
4.6 The SSpj family of algorithms 98
4.7 Matching the SSpj and GNpj algorithms 102
4.8 Particular examples of the SS$p2$ algorithms 114
4.9 The algorithm SS32B 133
4.10 Generalization of SS$p2$ 155
4.11 First order systems 156
4.12 Park's method 163
Exercises 166
References 167

5. Other single-step methods. Global crunch or divide and conquer?

5.1 Introduction 171
5.2 Methods using Hermitian interpolation 172
5.3 Rational approximation to the exponential 177
5.4 Runge–Kutta methods 193
5.5 Partitioning methods 205
5.6 Element by element methods 216
5.7 Operator splitting methods 222
Exercises 225
References 226

6. The energy method

6.1 Introduction 231
6.2 Newmark and the energy method 234
6.3 The energy method with Newmark including natural damping and forcing terms 238
6.4 The energy method with the SS22 algorithm 240
6.5 The energy method with Newmark and SS22 used on non-linear problems 241
6.6 The energy methods of Hughes and co-authors 248
6.7 Energy methods—more general algorithms 253
References 257

7. Non-linear equations

7.1 Introduction 259
7.2 Explicit methods, first and second order systems 262
7.3 *Ad hoc* iteration for first order systems 269
7.4 Lumped versus distributed mass matrices in subsurface flow 281

7.5 Newton or quasi-Newton method for first order non-linear equations 292
7.6 The non-linear second order dynamic vibration equation 296
7.7 Starting conditions 310
7.8 Time-step adjustment 313
7.9 Conjugate gradient methods 316
References 320

8. Coupled systems

8.1 Introduction 325
8.2 Monolithic schemes 326
8.3 Partitioning methods for coupled systems 336
8.4 Coupled first order and second order systems 342
8.5 Coupled systems of second order–second order equations—fluid–structure 350
References 353

Appendix 1. The Routh–Hurwitz conditions 356
References 357

Solutions to Exercises 358

Author Index 365

Subject Index 369