Contents

Preface ix Notes to the Instructor xii

Part I Linear Analysis and Matrices

Preview 1 Introduction 1

1 Fundamental Definitions 3

Classification of Equations 3

Linear Simultaneous Equations 4

Gaussian Elimination 8

Potential Problems in Gaussian Elimination 11

Matrix Notation and Multiplication 13

Arithmetic of Matrices 18

Multiplication of Row and Column Vector Matrices 19

- (§) Applications of Matrix Multiplication* 23
- (§) Markov Matrices 24
- (§) Rotation of Coordinate Systems 25
- (§) Gram-Schmidt Orthogonalization 29
- (§) Reciprocal Basis 32

Problems

Summary

MATLAB Functions Used

2 Solution of the Matrix Equation $\mathbf{A} ec{x} = ec{b}$ 52

Existence and Uniqueness of Solutions to $A\vec{x} = \vec{b}$ 53

LU Factorization of a Matrix 60

The Inverse Matrix 64

The Accuracy of a Solution 71

Iterative Solutions to Matrix Equations 75

MATLAB Functions Related to Matrix Inversion 78

Problems

Summary

MATLAB Functions Used

3 Matrix Eigenvalue Equations 90

Geometrical Transformations on Vectors in Two Dimensions 90
The Characteristic Equation and the Cayley-Hamilton Theorem 100
Reducing a Symmetric Matrix to Diagonal Form by Jacobi Rotations 102

^{*}Sections and chapters marked with (§) may be omitted without loss of continuity.

MATLAB Functions for Eigenvalue Problems 106 Perspective on Eigenvalue Equations 107

(§) Spectral Decomposition 112

(§) Singular Value Decomposition 115

MATLAB Functions Used Summary **Problems**

Bibliography for Part I 129

MATLAB Applied to Functions of a Single Variable Part II

Preview 131 Introduction 132

Series Expansions for Functions 133

Radius of Convergence of a Power Series 133

Acceleration of Convergence 135

Taylor Series 139

Padé Approximations 142

M-File Functions 145

Problems Summary **MATLAB Functions Used**

Roots of Functions of a Single Variable 151

Roots of Polynomials 152

Searching for Roots 154 Refining an Interval by Interval-Halving 156

Newton's Method for Finding Roots 158

The Secant Method 162

Summary **Problems**

MATLAB Functions Used

Minimum or Maximum of a Function

General Prescription for Finding a Minimum 170

Stepping Search for an Interval Containing a Minimum 171

Refining the Interval Containing a Minimum 172

MATLAB Functions Used **Problems** Summary

Numerical Integration Techniques 181

The Trapezoidal Rule for Integration 182

Simpson's Rule 185

Romberg Integration 191

The MATLAB Toolbox Functions quad and quad8 192

Gaussian Quadrature 194

Improper Integrals 198

Summary **Problems**

MATLAB Functions Used

Bibliography for Part II 205

Part III Functions of Two or More Variables

Preview 207 Introduction 208

B Derivatives of Multivariable Functions 209

Partial Derivatives 209

The Gradient 212

Taylor Series in Two Dimensions 218

Problems Summary

MATLAB Functions Used

9 Roots of Multivariable Functions 224

Zero Contours of a Function of Two Variables 224

Simultaneous Nonlinear Functions 227

Problems Summary

MATLAB Functions Used

10 (ξ) Minimization of a Multivariable Function 235

Simplex Methods for Finding Minima 235

Minimization Using Directional Methods 239

Quasi-Newton Method for Minimization of Multivariable Functions 242

Problems Summary

MATLAB Functions Used

11 Multiple Integrals 255

MATLAB Code for Iterated Integrals 255

Romberg Techniques for Double Integrals 260

Special Considerations Regarding Multidimensional Integrals 264

Problems

Summary MATLAB Functions Used

12 (§) Monte Carlo Integration 268

Integrals as an Average of a Function 268

Monte Carlo Integration Using Random Sampling 270

Monte Carlo Methods for Integrals with Complex Boundaries 274

Summary of the Monte Carlo Integration Method 279

Problems Summary

MATLAB Functions Used

Bibliography for Part III 282

Part IV (§) Data Analysis and Modeling

Preview 285

13 Statistical Description of Data 287

Moments of a Distribution 287 Modes of a Distribution 290

Higher Moments of a Distribution: The Skewness and the Kurtosis	292		
Distribution Functions 294			
Properties of Distribution Functions 298			
Combinations of Distribution Functions 299			

Comparison of Distributions

Problems MATLAB Functions Used Summary

Linear Least Squares Analysis 313

Systematic versus Random Errors 314 Least Squares Fitting of a Straight Line Linear Least Squares Fitting of Polynomials 321 General Linear Least Squares 323 Fitting with Legendre Polynomials **Problems** Summary MATLAB Functions Used

15 Nonlinear Least Squares 336

Pseudolinear Least Squares 336 **Elementary Nonlinear Methods 338** Use of the Toolbox Function fmins to Find the Optimum χ^2 More Sophisticated Nonlinear Methods 344 **Problems** Summary MATLAB Functions Used

16 Interpolation and Spline Fits 351

Lagrange Interpolation 351 Neville's Iterative Construction of the Lagrange Polynomial 354 **Bulirsch-Stoer Interpolation and Extrapolation 359**

Cubic Spline Interpolation 363

Problems Summary **MATLAB Functions Used**

17 Fourier Analysis

The Fourier Series 372 A Complex Version of the Fourier Expansion 374

The Fourier Transform 375

382 The Discrete Fourier Transform Fast Fourier Transforms (FFTs)

385 **Problems** Summary MATLAB Functions Used

Bibliography for Part IV

Part V **Differential Equations**

Preview 399

Elementary Concepts 401

First-Order Differential Equations 401 Second-Order Differential Equations 404

Replacing an	nth-Order Equation	by n Coupled First-Order Equations	405
Problems	Summary	MATLAB Functions Used	

19 Initial Value Differential Equations 408

Elementary Stepping Procedures for First-Order Equations 409
Improvements to Euler's Method 413
The Method of Runge-Kutta 417
MATLAB Implementation of Runge-Kutta Algorithms 419
(§) The Bulirsch-Stoer Extrapolation Method 424
Problems Summary MATLAB Functions Used

20 (§) Two-Point Boundary Value Problems 434

Shooting Methods for Boundary Value Problems 436
Relaxation Methods 442
The Finite Element Method 451
Problems Summary MATLAB Functions Used

21 (§) The Method of Superposition 463

Theoretical Properties of Linear Boundary Value Problems 463
The Superposition Method 466
Variation of Parameters 469
Problems Summary MATLAB Functions Used

(\S) Introduction to Partial Differential Equations 475

Preliminary Concepts 476
Initial Value Type Methods 482
Boundary Value Problems—Elliptic Equations 495
Beyond This Chapter 501
Problems Summary MATLAB Functions Used

Bibliography for Part V 508

Appendix Programming in MATLAB

A.i Matrix Algebra in MATLAB 513

Basic Arithmetic Operators 513
Entering Matrices and Vectors 514
Mathematical Functions Available in MATLAB 515
Referencing Elements of a Matrix 516
Special Matrices in MATLAB 518
Special MATLAB Functions for Constructing Large Matrices 519
The Colon for Characterizing Parts of a Matrix 520
Some Cautions When Entering Matrices in MATLAB 521
Problems Summary MATLAB Functions Used

A.ii Polynomials and Other Functions 526

Evaluation of Polynomials 526

Loop Structures in MATLAB 527

Vectorized Arithmetic Operations 530

Multiplication and Division of Polynomials 532

Elementary MATLAB M-Files and Polynomials 534

Writing M-files with an Editor 537

Problems Summary MATLAB Functions Used

A.iii MATLAB Logical Structures 545

MATLAB Logical Structures 545
MATLAB if Structures 550
Use of the MATLAB Function feval

Problems Summary MATLAB Functions Used

552

A.iv Two-Dimensional Plotting 558

The MATLAB Plotting Function plot(x,y) 558
Printing a Graph 561

Problems Summary MATLAB Functions Used
MATLAB Functions 565

A.v Constructing and Plotting Multivariable Functions 567

Establishing a Grid of x,y Values 567
Three-Dimensional Perspective Graphs 570
Contour Graphs 577
Non-Cartesian Coordinates 578
Plotting Three-Dimensional Trajectories 582
Four-Dimensional Plots 583
Problems Summary MATLAB Functions Used

A.vi Elementary Input/Output in MATLAB 590

Connecting a Data File to MATLAB: The fopen Statement 591
Reading Data from an Existing File: The fscanf Statement 591
Output from MATLAB Using fprintf 593
Problems MATLAB Functions Used

Answers 597

Index 629