

# Contents

<b>1</b>	<b>Introduction</b>	<b>15</b>
1.1	Getting Started.....	15
	General Setting .....	15
	The Startresidual .....	16
	An Isomorphism .....	16
	An Approximation Problem .....	17
	An Inner Product .....	18
	MATLAB/MATHEMATICA Implementations .....	18
<b>2</b>	<b>Orthogonal Polynomials</b>	<b>19</b>
2.1	General Properties.....	19
	Basic Definitions .....	19
	Three-Term Recurrence Relation .....	23
	Stieltjes Procedure .....	26
	Jacobi Matrix .....	27
	Extremal Property of Orthogonal Polynomials .....	28
	Zeros of Orthogonal Polynomials .....	29
	Computing Zeros of Orthogonal Polynomials .....	30
2.2	Some Applications.....	30
	Gaussian Quadrature .....	30
	Moments and Distribution Functions .....	34
	Associated Polynomials and Continued Fractions .....	37
2.3	A Useful Tool .....	44
	QR Factorization; Tridiagonal Case .....	44
2.4	Orthogonal Residual Polynomials .....	48
	Recurrence Relations for Orthogonal Residual Polynomials .....	48
	Extremal Property of Orthogonal Residual Polynomials .....	51
2.5	Kernel Polynomials .....	55
	Definition and Orthogonality .....	55
	Extremal Property of Kernel Polynomials .....	56

Recurrence Relations for Kernel Polynomials .....	60
Zeros of Kernel Polynomials .....	65
Computing Zeros of Kernel Polynomials .....	69
2.6 Hermite Kernel Polynomials .....	70
Definition and Orthogonality .....	71
Extremal Property and a Christoffel-Darboux Formula .....	73
Recurrence Relations for Hermite Kernel Polynomials .....	79
Zeros of Hermite Kernel Polynomials .....	83
2.7 Orthogonal and (Hermite) Kernel Polynomials .....	84
The ME Connection .....	85
The MR Connection .....	87
The ME - MR Connection .....	89
<b>3 Chebyshev and Optimal Polynomials</b>	<b>90</b>
3.1 Basic Definitions.....	90
Green's Function .....	92
Equilibrium Distribution .....	94
Characterization of the Best Approximation .....	96
3.2 Chebyshev and Optimal Polynomials; One Interval .....	97
3.3 Chebyshev and Optimal Polynomials; Two Intervals.....	102
Basic Observations .....	102
One More Extremal Point .....	103
Elliptic Functions .....	110
A Conformal Mapping .....	115
Green's Function for the Union of Two Disjoint Intervals .....	119
The Achieser Representation of the Chebyshev Polynomials ..	121
3.4 Computing an Asymptotic Convergence Factor.....	125
The Inverse of the Elliptic Sine .....	126
MATLAB Implementation of SNINV .....	128
Evaluation of a Theta Function .....	129
MATLAB Implementation of ASYMPFAC .....	130
<b>4 Orthogonal Polynomials and Krylov Subspaces</b>	<b>132</b>
4.1 Generating a Basis; Orthonormal Case.....	132
Lanczos Method .....	134
MATLAB Implementation of LANCZOS .....	134
4.2 Generating a Basis; Monic Case.....	135
<b>5 Estimating the Spectrum and the Distribution function</b>	<b>137</b>

5.1	The Model Problem .....	137
5.2	Estimating the Spectrum .....	140
5.3	Approximating the Distribution Function .....	144
	Lanczos Method and Distribution Functions .....	145
	Monotone Spline .....	149
	MATLAB Implementation of MPCI .....	151
	Computing New Orthogonal Polynomials .....	152
<b>6</b>	<b>Parameter Free Methods</b>	<b>155</b>
6.1	Overview .....	155
6.2	Implementations Based on Three - Term Recurrences .....	159
	Basic Algorithm .....	159
	The CG Approach .....	161
	MATLAB Implementation of CG .....	162
	The CR Approach .....	164
	MATLAB Implementation of CR .....	165
6.3	CG/CR Applied to Indefinite Systems .....	166
6.4	Implementations Based on the Monic Basis .....	173
	The STOD Approach .....	174
	MATLAB Implementation of STOD .....	175
	The MCR Approach .....	176
	MATLAB Implementation of MCR .....	177
	Modifications .....	178
6.5	Implementations Based on the Lanczos Basis .....	179
	The SYMMLQ Approach .....	180
	MATLAB Implementation of SYMMLQ .....	183
	The MINRES Approach .....	185
	MATLAB Implementation of MINRES .....	187
6.6	Residual Smoothing .....	188
6.7	A “Non-Feasible” Approach .....	189
	MATHEMATICA Computation of the Minimal Error .....	190
6.8	Implementations Based on Normal Equations .....	191
	The Golub/Kahan Bidiagonalization .....	192
	QR Factorization; Bidiagonal Case .....	194
	The LSQR Approach .....	195
	MATLAB Implementation of LSQR .....	197
	The CRAIG Approach .....	199

MATLAB Implementation of CRAIG .....	201
6.9 Comparison of the Various Methods .....	202
Symmetric Spectrum .....	207
<b>7 Parameter Dependent Methods</b>	<b>212</b>
7.1 The Chebyshev Iteration for Symmetric Indefinite Systems .....	212
7.2 Methods Based on the Eigenvalue Distribution .....	217
<b>8 The Stokes Problem</b>	<b>224</b>
8.1 The Continuous Problem .....	224
Hilbert Spaces .....	225
The Continuous Stokes Problem .....	226
Variational Formulation .....	227
Saddle Point Problems .....	229
Existence and Uniqueness of Solutions .....	230
8.2 The Discrete Problem .....	231
The Linear System .....	234
8.3 Some Finite Element Spaces .....	237
<b>9 Approximating the A-Norm</b>	<b>248</b>
9.1 Energy Norm .....	248
9.2 Approximating the A-Norm of the Error .....	250
CG Case .....	250
MATLAB Implementation of cfAerr .....	256
Two Examples .....	256
General Case .....	259
Lower and Upper Bounds .....	262
<b>10 Bibliography</b>	<b>263</b>
<b>11 Notation</b>	<b>274</b>
<b>12 Index</b>	<b>278</b>