

# Contents

<b>1</b>	<b>Necessities and Criteriaions</b>	<b>11</b>
<b>2</b>	<b>Introduction to Iterative Methods</b>	<b>14</b>
2.1	Iterates, Residuals and Errors . . . . .	14
2.2	Classical Iterative Methods . . . . .	16
2.2.1	Matrix Splittings . . . . .	17
2.2.2	Method of Steepest Descent . . . . .	19
2.2.3	Classical Conjugate Gradients (CG) . . . . .	21
2.2.4	Review of Classical Iterative Methods . . . . .	27
<b>3</b>	<b>Orthogonalization Methods</b>	<b>28</b>
3.1	General Concept . . . . .	28
3.2	Convergence and Geometric Properties . . . . .	31
3.3	Matrix Notation and Short Recurrences . . . . .	39
<b>4</b>	<b>Krylov Subspace Methods</b>	<b>43</b>
4.1	Conjugate Krylov Subspace Methods (CKS) . . . . .	44
4.1.1	The Lanczos Connection . . . . .	45
4.1.2	Convergence Properties . . . . .	54
4.1.3	Short Recurrences . . . . .	59
4.1.4	Generalized Conjugate Gradient Methods . . . . .	62
4.1.5	Biconjugate Gradient-based Methods . . . . .	70
4.1.6	Rank- $j$ Updates . . . . .	94
4.1.7	Overview of CKS methods . . . . .	97
4.1.8	Poly- and Hybrid Algorithms . . . . .	101
4.2	Generalized Minimal Error Methods . . . . .	102
4.3	Broyden and Eirola-Nevanlinna Methods . . . . .	107
4.4	Classification in Terms of Orthogonalization Methods . . . . .	108
4.4.1	Special Methods . . . . .	108
4.4.2	Relations between Methods . . . . .	115
4.5	A Sample Problem . . . . .	116
4.6	Discussion . . . . .	121

<b>5</b>	<b>Transformation of Methods</b>	<b>131</b>
5.1	Residual-Minimizing Smoothing . . . . .	132
5.2	Quasi-Residual Smoothing . . . . .	139
5.3	Error-Minimizing Smoothing . . . . .	141
5.4	Review and Comparison of Smoothing . . . . .	143
<b>6</b>	<b>Preconditioning</b>	<b>145</b>
6.1	Constant Preconditioning . . . . .	145
6.2	Step-Dependent Preconditioning . . . . .	147
6.3	Preconditioned Orthogonalization Methods . . . . .	150
6.4	Overview of Preconditioners . . . . .	154
6.5	Special Preconditioners . . . . .	154
6.5.1	Normalization . . . . .	155
6.5.2	Polynomial Preconditioners . . . . .	158
6.5.3	Normal Equations . . . . .	159
6.5.4	Incomplete <i>LU</i> -Decomposition (ILU) . . . . .	159
6.5.5	Incomplete Cholesky (IC) . . . . .	161
6.5.6	Incomplete <i>QR</i> -Decomposition (IQR) . . . . .	162
6.5.7	Incomplete Gauss-Jordan . . . . .	163
6.5.8	Incomplete Frobenius Inverse . . . . .	165
6.5.9	Projection Methods . . . . .	166
6.5.10	Iterative Methods as Preconditioners . . . . .	167
6.5.11	Multi-Level Methods . . . . .	167
6.6	Discussion . . . . .	171
<b>7</b>	<b>Numerical Stability</b>	<b>174</b>
7.1	Arithmetic and Accuracy . . . . .	174
7.2	Variants and Different Implementations . . . . .	175
<b>8</b>	<b>Efficiency on Supercomputers</b>	<b>178</b>
8.1	Vectorization and Parallelization . . . . .	178
8.2	Latency Hiding . . . . .	179
8.3	Scalability . . . . .	179
8.4	Matrix-Vector Multiplication . . . . .	180
8.5	Dot Products . . . . .	182
<b>9</b>	<b>Remarks and Annotations</b>	<b>185</b>
9.1	Review and Practical Advices . . . . .	185
9.2	Open Questions and Trends . . . . .	187

---

<b>Time Table</b>	<b>190</b>
<b>Notation</b>	<b>191</b>
<b>Bibliography</b>	<b>194</b>
<b>Index</b>	<b>214</b>