

Contents

CHAPTER 1. FINITE SYSTEMS 1

- 1.1. Vectors and Linear Spaces 1
- 1.2. Matrices and Linear Transformations 12
- 1.3. Ordinary Differential Systems 19
- 1.4. Finite Mechanical Systems 26
- 1.5. Lagrange's Equations and Hamilton's Principle 28
- 1.6. Systems with Constant Coefficients 35
- 1.7. The Response Matrix and Distributions 39
- 1.8. Two-Point Boundary Conditions 44

CHAPTER 2. DISTRIBUTIONS AND WAVES 49

- 2.1. Equations in Two Independent Variables 49
- 2.2. Wave Motion of a String 53
- 2.3. Reflection of Waves 59
- 2.4. Theory of Distributions 66
- 2.5. Applications to the Initial Problem 73
- 2.6. The Separation of Variables 81
- 2.7. Fourier Series 87

CHAPTER 3. PARABOLIC EQUATIONS AND FOURIER INTEGRALS 95

- 3.1. The Heat Flow Equation 95
- 3.2. Heat Flow on a Finite Interval 100
- 3.3. Fourier Integral Transforms 108
- 3.4. Diffusion on an Infinite Interval 113
- 3.5. Semi-Infinite Intervals 118
- 3.6. Fourier Transforms of Distributions 124
- 3.7. Finite Difference Calculations 129

CHAPTER 4. LAPLACE'S EQUATION AND COMPLEX VARIABLES 133

- 4.1. Mathematical and Physical Applications 133
- 4.2. Boundary Value Problems for Harmonic Functions 136
- 4.3. Circular Harmonics 141
- 4.4. Rectangular Harmonics 148
- 4.5. Half-Plane Problems 153
- 4.6. Complex Integrals 159
- 4.7. Fourier and Laplace Transforms 167
- 4.8. The Finite Difference Laplace Equation 173

CHAPTER 5. EQUATIONS OF MOTION 180

- 5.1. Vibrations of a Membrane 180
- 5.2. Lateral Vibration of Rods and Plates 187
- 5.3. Integral Theorems and Vector Calculus 194
- 5.4. Equations of Motion of an Elastic Solid 201
- 5.5. Motion of a Fluid 206
- 5.6. Equations of the Electromagnetic Field 214
- 5.7. Equations of Quantum Mechanics 220

CHAPTER 6. GENERAL THEORY OF
EIGENVALUES AND EIGENFUNCTIONS 228

- 6.1. The Minimum Problem 228
- 6.2. Sequences of Eigenvalues and Eigenfunctions 233
- 6.3. Variational Properties of Eigenvalues and Eigenfunctions 239
- 6.4. Eigenfunction Expansions 244
- 6.5. The Rayleigh-Ritz Approximation Method 250
- 6.6. On the Separation of Variables 254
- 6.7. Series Expansions and Integral Transforms 260

CHAPTER 7. GREEN'S FUNCTIONS 264

- 7.1. Inverses of Differential Operators 264
- 7.2. Examples of Green's Functions 270
- 7.3. The Neumann and Robin Functions 277
- 7.4. Differential and Integral Equations 283
- 7.5. Source Functions for Parabolic Equations 288
- 7.6. Convergence of Series of Distributions 292

CHAPTER 8. CYLINDRICAL EIGENFUNCTIONS 298

- 8.1. Bessel Functions 298
- 8.2. Eigenfunctions for Finite Regions 306
- 8.3. The Fourier-Bessel Series 310
- 8.4. The Green's Function 314
- 8.5. Functions of Large Argument 317
- 8.6. Diffraction by a Cylinder 323
- 8.7. Modified Bessel Functions 327
- 8.8. The Hankel and Weber Formulas 333

CHAPTER 9. SPHERICAL EIGENFUNCTIONS 341

- 9.1. Legendre Functions 341
- 9.2. Eigenfunctions of the Spherical Surface 345
- 9.3. Eigenfunctions for the Solid Sphere 351
- 9.4. Diffraction by a Sphere: Addition Theorem 356
- 9.5. Interior and Exterior Expansions 362
- 9.6. Functions of Nonintegral Order 367

CHAPTER 10. WAVE PROPAGATION IN SPACE 372

- 10.1. Characteristic Surfaces 372
- 10.2. Source Function for the Wave Equation 378
- 10.3. Applications. Huyghens' Premise 385
- 10.4. Electromagnetic and Elastic Waves 390
- 10.5. Wave Fronts and Rays 397
- 10.6. Reflection and Diffraction 401

TABLES 411

BIBLIOGRAPHY 417

INDEX 419