

Contents

Chapter 1. Recurrence Relations and Iterative Methods

1.1	First Order Linear Recurrence Relations	1
1.2	Second Order Linear Recurrence Relations	7
1.3	Non-linear Recurrence Relations for Solving $f(x)=0$	13
1.4	Recurrence Relations Involving Matrices	22
1.4.1	The Conditioning of the Linear Equations $Ax=b$	24
1.4.2	Gaussian Elimination and Partial Pivoting	28
1.4.3	Tridiagonal Matrices	30
1.4.4	Iterative Refinement	34
1.4.5	The Iterative Solution of Linear Equations	35
1.4.6	Recurrence Methods for the Determination of Eigenvalues	38
1.5	Extrapolation and Acceleration of Convergence	40
1.5.1	Acceleration of Linearly Convergent Methods for $f(x)=0$	41
1.5.2	Acceleration Methods for Linear Equations	43
1.5.3	General Extrapolation Techniques	45

Chapter 2. Ordinary Differential Equations Initial Value Problems

2.1	Introduction	51
2.2	Simple Difference Methods	56
2.3	General Linear Single Step Methods	64
2.4	General Single Step Methods	65
2.5	Runge-Kutta Methods	70
2.6	Stability of Single Step Methods	76
2.7	General Linear Multi-step Methods	79
2.8	Predictor-Corrector Methods	95
2.9	Extrapolation - Improvement of Solution	100
2.10	Choice of Method	103
2.11	Higher Order and Systems of Equations of Initial Value Type	106
2.12	Special Methods for Initial Value Problems of Second Order	114

Chapter 3. Ordinary Differential Equations Boundary Value Problems

3.1	Introduction	121
3.2	Simple Shooting Methods	123
3.3	Multiple Shooting Methods	132
3.4	Finite Difference Methods	135
3.4.1	Improvement of the Computed Solution	145
3.5	Finite Element Methods	152
3.5.1	The Method of Collocation	153
3.5.2	The Galerkin Method	157
3.5.3	The Rayleigh-Ritz Method	160
3.5.4	Non-Linear Problems	161

Chapter 4. Parabolic Partial Differential Equations

4.1	Introduction	163
4.2	Simple Numerical Methods for Solving the Heat Equation $u_t = \sigma u_{xx}$	166
4.3	Implicit Methods for Solving the Heat Equation $u_t = \sigma u_{xx}$	178
4.4	Generalisations	184
4.5	An Introduction to Non-linear Equations	189
4.6	Equations Involving More Than One Spatial Variable	191

Chapter 5. Hyperbolic Partial Differential Equations

5.1	Introduction	197
5.2	Simple Difference Methods for First Order Hyperbolic Equations	202
5.2.1	Variable Coefficients	212
5.2.2	Inhomogeneous First Order Hyperbolic Equations	213
5.2.3	Non-linear First Order Hyperbolic Equations	214
5.2.4	Systems of Hyperbolic Equations	215

Chapter 5. Hyperbolic Partial Differential Equations (Ctnd)

5.3	The Method of Characteristics	216
5.4	Second Order Hyperbolic Equations	219
5.4.1	Finite Difference Methods	220
5.4.2	The Method of Characteristics	222
5.4.3	The Wave Equation	224
5.5	Concluding Remarks Regarding Hyperbolic Equations	226

Chapter 6. Elliptic Partial Differential Equations

6.1	Introduction	229
6.2	Simple Methods for Laplace's Equation on $0 < x, y < 1$	231
6.3	Generalisations	240
6.4	Neumann Problems	243
6.5	Singularities	247
6.6	Finite Element Methods	251

Appendix 1.	The Algebraic Eigenvalue Problem	261
--------------------	---	-----

Appendix 2.	The Classification of Partial Differential Equations	265
--------------------	---	-----

Solutions for Exercises in Chapter 1	271
---	-----

Literature	273
-------------------	-----

Index	279
--------------	-----