

CONTENTS

SURVEY		1
CHAPTER 1 Stiff differential equations		5
1.1	Stiffness	5
1.2	The one-sided Lipschitz condition	15
1.3	Dissipativity	17
1.4	The linear model system	17
1.5	The logarithmic matrix norm	27
1.6	Remarks	34
CHAPTER 2 Contractivity and stability		37
2.1	Some definitions on contractivity and stability	37
2.2	Absolute stability	40
2.3	Unconditional contractivity for the linear model system	43
2.4	A specific property of implicit Euler	46
2.5	The à priori global error bound	48
2.6	An instructive example	51
CHAPTER 3 Runge-Kutta methods		55
3.1	The general structure of a Runge-Kutta formula	55
3.2	The simplifying conditions	57
3.3	Methods based on high order quadrature	62
3.4	The stability function	71
3.5	Diagonally and singly-implicit methods	75
3.6	The W-transformation	83
3.7	The direct product of matrices	93
CHAPTER 4 Contractivity of Runge-Kutta methods		97
4.1	B-stability	97
4.2	Algebraic stability	102
4.3	Relations between various stability properties	104
4.4	Reducibility	107
4.5	Reducibility and stability	114
4.6	Algebraically stable methods	117

CHAPTER 5 Solution of the algebraic equations in Runge-Kutta schemes	131
5.1 Notational conventions	132
5.2 Existence and uniqueness for the linear model system	136
5.3 Existence and uniqueness for nonlinear problems	143
5.4 BSI-stability	151
5.5 BSI-stability of Gauss-Legendre methods	155
5.6 BSI-stability of Radau IA-methods	158
5.7 BSI-stability of Radau IIA-methods	162
5.8 BSI-stability of Lobatto IIIA- and IIIB-methods	164
5.9 BSI-stability of Lobatto IIIC-methods	169
5.10 BSI-stability of diagonally and singly-implicit methods	172
5.11 Generalizations of B-stability	177
5.12 Implementation of implicit Runge-Kutta methods	180
CHAPTER 6 Contractivity of explicit methods	185
6.1 The model class	185
6.2 The concept of circle contractivity	187
6.3 Examples of circle contractive methods	191
CHAPTER 7 The concept of B-convergence	195
7.1 The model problem of Prothero and Robinson	196
7.2 B-consistency and B-convergence	199
7.3 Sufficient conditions for B-consistency	204
7.4 Sufficient conditions for B-convergence	209
7.5 A numerical illustration	212
7.6 Remarks	216
CHAPTER 8 The concept of D-stability	219
8.1 The definition of D-stability	219
8.2 The problem class \mathfrak{S}	220
8.3 D-stability of implicit Euler	225
8.4 On D-stability and stepwise stability	227
CHAPTER 9 Runge-Kutta Rosenbrock methods	231
9.1 The variation of constants formula	232
9.2 Stability of the variation of constants formula	234
9.3 Results on D-stability	237

9.4	On the stability of implemented Runge-Kutta schemes	245
9.5	D-stability and the autonomous form	249
9.6	Some remarks on B-consistency and B-convergence	251
9.7	A numerical illustration	252
CHAPTER 10 Applications to partial differential equations		255
10.1	The method of lines	255
10.2	Review of some stability concepts for semi-discrete problems	257
10.3	Review of some stability concepts for integration formulas	261
10.4	A pseudo-linear parabolic problem	269
10.5	The hyperbolic model problem	272
10.6	Diffusion-convection problems	275
10.7	The shallow water equations: conservative space differencing	277
10.8	The shallow water equations: conservative time integration	286
BIBLIOGRAPHY AND AUTHOR INDEX		289
SUBJECT INDEX		301
SYMBOL INDEX		306