Contents

Preface xi

Foreword 1

Chapter 1. The Time-Frequency Problem 9

1.1. The Time-Frequency Duality and Its Bars 10

1.1.1. Fourier Analysis 10

Limitations 10. *Citations* 11.

1.1.2. Heisenberg-Gabor Uncertainty Principle 12

The time-frequency inequality 14.
Interpretations 16.

1.1.3. Slepian-Pollak-Landau Theory 18

Concentrations 18. *Sampling* 19.
The eigenvalue equation 21.
Approximation of bandlimited signals 23.
Approximative dimension of a signal 24.
Inequality of the concentrations 24.

1.2. Leaving Fourier? 26

1.2.1. Local Quantities 26

Instantaneous frequency 27. *Group delay* 31.
Interpretative remarks 31. *An example* 32.

1.2.2. Nonstationary Signals 36

Definition 36. *Generalizations* 38. *Variations* 40.

1.3. Towards Time-Frequency: Several Approaches 42

1.3.1. The Time-Frequency Plane and Its Three Readings 43

Frequency (time) 43. *Time (frequency)* 43.
Time-frequency 43.

1.3.2. Decompositions, Distributions, Models 43

Decompositions 44. *Distributions* 44. *Models* 44.

1.3.3. Moving and Joint, Adaptive and Evolutionary Methods 45

Moving and joint 45. *Adaptive and evolutionary* 45.

Chapter 1 Notes 46

Chapter 2. Classes of Solutions 49

2.1. An Introduction with Historical Landmarks 50

2.1.1. Short-Time Fourier and Instantaneous Spectrum 50

Sonagram and spectrogram 51. *Restrictions* 52.

2.1.2. Atomic Decompositions 54

Gabor 54. *Variations* 55. *Wavelets* 56.

2.1.3. Pseudo-Densities 57

Wigner-Ville 58. *Page* 60. *Rihaczek* 60.
Extensions 62. *Unification* 62.

2.1.4. The Parallel to Quantum Mechanics 64

Different concerns 64. *Some intersections* 65.

2.2. Atomic Decompositions 66

2.2.1. Projections and Bases - General Principles 66

Discrete bases 66. *Continuous bases* 67. *Frames* 69.

2.2.2. Time-Frequency Examples 70

Short-time Fourier 70. *Obstruction established by*
Balian–Low Theorem 73. *Gabor and variants* 75.

2.2.3. Time-Scale Examples 76

Continuous wavelets 76. *Discrete wavelets* 80.
Multiresolution analyses and orthonormal bases 83.
Pyramidal algorithms 89. *Some wavelet bases* 92.

2.2.4. A "Detection-Estimation" Viewpoint 97

Ambiguity functions 97. *Atoms and matched filtering* 101.

2.3. The Energy Distributions 103

General setting 103. *Covariance principles* 104.

2.3.1. Construction of the Bilinear Classes 104

Time-frequency 104. *Time-scale* 107.

2.3.2. The Troika of Parameterizations-Definitions-Properties ... 109

Definitions 110. *Constraints* 114. *Cohen's class* 116.
Affine class 132.

2.3.3. Results of Exclusion and Conditional Uniqueness 142

Wigner's Theorem 143. *Some results of exclusion* 145.
Some results on conditional uniqueness 147.

2.4. The Power Distributions 150
2.4.1. From Deterministic to Random Signals 150
Decompositions and fluctuations 150.
Distributions and expectation values 151.
Cramér and beyond 152.
2.4.2. The Orthogonal (or Almost Orthogonal) Solutions 153
Karhunen decompositions 153.
Priestley spectrum 154.
Tjøstheim, Mélard, Grenier approach 156.
2.4.3. The Frequency Solutions 160
Harmonizable signals 160.
Wigner-Ville spectrum 165.
2.4.4. Some Links Between the Different Spectra 170
Continuous time 170. *Discrete time* 171.
Chapter 2 Notes 174

Chapter 3. Issues of Interpretation 183
3.1. About the Bilinear Classes 185
3.1.1. The Different Parameterizations 185
Time-frequency 187. *Time-time* 188.
Frequency-frequency 191. *Frequency-time* 191.
3.1.2. Parameterizations, Operators and Correspondence Rules ..194
Why operators? 194.
The operator of time-frequency shifts 195.
Correspondence rules 197. *Kernels* 199.
Weyl calculus 200. *Moments* 201.
Dilations and ambiguities 205.
3.1.3. Time-Frequency or Time-Scale? 208
Fourier scale 208. *Mellin scale* 210.
Analysis and decision statistics 213.

3.2. The Wigner-Ville Distribution and Its Geometry 213
3.2.1. Wigner-Ville versus Spectrogram 213
Structure of the distributions 213.
Pseudo-Wigner-Ville 215.
Supports 216. *Localization to chirps* 217.
Spectrogram and reassignment 221.
Discretization 225.
3.2.2. The Mechanism of Interferences 226
Construction principle 228.
A different perspective from the ambiguity plane 231.
Inner and outer interferences 232.
Approximation by the method of stationary phase 234.
Singularities and catastrophes 237.
Interferences, localization, and symmetries 243.
Generalization to the s-Wigner distributions 245.
Generalization to the affine distributions 247.
3.2.3. Reduction of the Interferences 252
Analytic signal 252.
Wigner-Ville and atomic decompositions 252.
Smoothing 254. *Coupled smoothing* 255.
Separable smoothing 256. *Joint smoothing* 261.
Variable and/or adapted smoothing 269.
"Image" approaches 274.
3.2.4. Usefulness of the Interferences 274
Unitarity 274. *Phase information* 275.
What is a component? 277.
3.2.5. Statistical Estimation of the Wigner-Ville Spectrum 279
Assumptions 280. *Classes of estimators* 281.
Bias 282. *Variance* 283. *Examples* 284.
3.3. About the Positivity 289
3.3.1. Some Problems Caused by the Nonpositivity 289
3.3.2. Positivity by the Signal 293
An example 293. *Hudson's theorem* 294.
Random signals and positive spectra 295.
3.3.3. Positivity by the Distribution 296
Positive distributions 296. *Positive smoothing* 297.
A stochastic interpretation 300.
Chapter 3 Notes 301

Chapter 4. Time-Frequency as a Paradigm 309
4.1. Localization 311
4.1.1. Heisenberg-Gabor Revisited 311
Example 1 311. *Example 2* 313.
4.1.2. Energy Concentration 315
Problem formulation 315.
The general eigenvalue equation 316.
Restriction to ellipsoidal domains 316.
Interpretations and conjecture 323.
4.1.3. Other Time-Frequency Inequalities 323
L_p*-norms* 324. *Localization and stationary phase* 325.
4.2. Signal Analysis 327
4.2.1. Time-Frequency, Time-Scale, and Spectral Analysis 327
Paving and marginal distributions 328.
The example of "$1/f$*-noise"* 329.
Analysis of self-similar processes 330.
4.2.2. Nonstationary Characteristics 334
Distance from the stationary case 334.
Demodulation 336. *Local singularities* 338.
Evolutionary singularities 340.
4.3. Decision Statistics 342
4.3.1. Matched Time-Frequency Filtering 343
4.3.2. Maximum Likelihood Estimators for Gaussian Processes .. 345
Classical solution 345.
Time-frequency formulation 346.
4.3.3. Some Examples 347
Rayleigh channel 347.
Detection of chirps and Doppler tolerance 347.
Locally optimal detection 349.
Time-frequency jitter 350.
A broader class of time-frequency receptors 351.
Chapter 4 Notes 354

Bibliography 359

Index 381