Table of Contents

Authors' Preface 13

Chapter 1 Basic mathematics

1.1 Introduction. 15
1.2 Sets of numbers 15
1.3 Equality and inequality 16
1.4 Significant figures and decimal places 17
1.5 The modulus of a real number 18
1.6 Factorial 19
1.7 Reciprocal 19
1.8 Rules of arithmetic 20
1.9 Indices 21
1.10 Some worked examples 22
1.11 Logarithms. 24
1.12 Graphs 25
1.13 The trigonometric ratios. 28
1.14 Trigonometric formulae 31
1.15 The inverse trigonometric functions 32
1.16 Summary and further reading 35
Problems 35

Chapter 2 Sequences and series

2.1 Introduction. 43
2.2 Sequences and series 43
2.3 The arithmetic series 45
2.4 The geometric series. 47
2.5 The binomial theorem for integral index 50
2.6 The binomial theorem for general index. 54
Problems 57

Chapter 3 Polynomials

3.1 Introduction 63
3.2 Definition 63
3.3 Polynomial of degree one 64
3.4 Polynomial of degree two 66
3.4.1 Properties of complex numbers 68
3.4.2 Curve sketching 70
3.5 Polynomial of degree three 73
3.6 Solution of polynomial equations 75
3.7 Manipulation of polynomials 77
3.8 Interpolation 78
3.9 Summary 80
Problems 80

Chapter 4 Functions

4.1 Introduction 87
4.2 Functions 87
4.3 Inverse functions 88
4.4 Continuity 91
4.5 Limits 94
4.6 A trigonometric limit 97
Problems 99

Chapter 5 Differentiation

5.1 Introduction 105
5.2 The derivative 106
5.3 Derivative of a constant 110
5.4 Derivative of x^α 110
5.5 Derivatives of $\sin x$ and $\cos x$ 112
5.6 Rules of differentiation 113
5.7 Implicit differentiation 121
5.8 The inverse rule 122
5.9 Parametric differentiation 124
5.10 Higher derivatives 124
Problems 125

Chapter 6 Applications of differentiation

6.1 Introduction 133
6.2 Maxima and Minima 133
6.3 Curve sketching 136

6.4 Small changes 142
6.5 Related rates of change 144
6.6 L'Hospital's rule 145
Problems 145

Chapter 7 Maclaurin series
7.1 Introduction 151
7.2 Maclaurin series 151
7.3 Summary 156
Problems 156

Chapter 8 The solution of equations
8.1 Introduction 159
8.2 Trigonometric equations 159
8.3 Graphical methods 160
8.4 Newton's method 162
8.5 Summary 165
Problems 165

Chapter 9 Functions of several variables
9.1 Introduction 167
9.2 Differentiation of a function of two variables 167
9.3 Higher derivatives 170
9.4 More than two variables 172
9.5 Summary 173
Problems 173

Chapter 10 Integration I. The indefinite integral
10.1 Introduction 175
10.2 Integration 175
10.3 Standard integrals 176
10.4 Integrating a rate of change 179
10.5 Change of variable 182
10.6 Integration by parts 186
Problems 189

Chapter 11 Integration II. The definite integral
11.1 Introduction 193
11.2 Area under a curve 193

11.3 The definite integral . . . 196
11.4 Change of variable in a definite integral . . . 199
11.5 Differentiation with respect to a limit of integration . . . 202
11.6 The definite integral as an infinite sum . . . 202
Problems . . . 204

Chapter 12 Logarithmic and exponential functions
12.1 Introduction . . . 209
12.2 The logarithmic function . . . 209
12.3 Properties of $\ln x$ . . . 211
12.4 The exponential function . . . 218
Problems . . . 224

Chapter 13 The integration of rational polynomials
13.1 Introduction . . . 231
13.2 Denominator quadratic . . . 231
13.2.1 Completing the square . . . 232
13.2.2 Partial fractions . . . 234
13.3 Denominator cubic . . . 236
Problems . . . 237

Chapter 14 Applications of integration
14.1 Introduction . . . 241
14.2 Area between two curves . . . 241
14.3 Volumes . . . 247
14.4 Length of arc of a plane curve . . . 250
14.5 Area of surface of revolution . . . 252
14.6 Mean values . . . 252
Problems . . . 253

Chapter 15 Differential equations
15.1 Introduction . . . 257
15.2 First order differential equations . . . 259
15.2.1 Variables separable differential equations . . . 259
15.2.2 Population models . . . 263
15.2.3 Epidemic models . . . 267
15.2.4 Linear differential equations . . . 268
15.3 Linear constant coefficient differential equations . . . 272
15.4 Systems of first order differential equations . . . 277
Problems . . . 281

Chapter 16 Recurrence equations

16.1 Introduction 291
16.2 First order recurrence equations 292
16.3 Linear constant coefficient recurrence equations 295
Problems 302

Chapter 17 Numerical methods

17.1 Introduction 307
17.2 Errors 308
17.3 Roots of equations 310
17.4 The evaluation of definite integrals 314
17.4.1 Trapezoidal rule 317
17.4.2 Simpson's rule 318
17.4.3 Integrals with infinite limits of integration 323
17.5 The numerical solution of first order ordinary differential equations 327
Problems 333

Chapter 18 Matrices

18.1 Introduction 339
18.2 Matrices 339
18.3 The algebra of matrices 340
18.3.1 Population matrices 346
18.4 The transposed matrix 348
18.5 The inverse matrix 349
18.6 Solving simultaneous linear equations by Gaussian elimination . . . 351
18.7 Finding the inverse of a matrix 354
18.8 Determinants 356
18.9 Eigenvalues and eigenvectors 358
18.10 Numerical methods 362
Problems 365

Chapter 19 Curve fitting

19.1 Introduction 375
19.2 Linear relationships 375
19.3 Power laws 379
19.4 Exponential laws 382
19.5 The method of least squares 384
Problems 388

Chapter 20 Permutations and combinations

20.1 Introduction . . . 395
20.2 Performing successive operations . . . 395
20.3 Permutations and combinations . . . 396
20.4 Like and unlike objects . . . 398
Problems . . . 399

Chapter 21 Probability

21.1 Introduction . . . 401
21.1.1 Historical probability . . . 402
21.1.2 Probability when the outcomes are equally likely . . . 402
21.2 Laws of probability . . . 404
21.3 Applications of the probability laws . . . 407
Problems . . . 412

Chapter 22 Populations and probability distributions

22.1 Introduction . . . 415
22.2 Describing a population . . . 415
22.3 Interpreting probability distributions . . . 420
22.3.1 Calculating the mean and variance . . . 422
22.4 Binomial distribution . . . 425
22.4.1 Mathematical basis of the binomial distribution . . . 427
22.4.2 Mean and variance of the binomial distribution . . . 428
22.5 Poisson distribution . . . 430
22.5.1 Mean and variance of the Poisson distribution . . . 431
22.6 Normal distribution . . . 433
22.7 Approximation of the binomial distribution by the normal distribution . . . 436
Problems . . . 438

Chapter 23 Samples and sampling distributions

23.1 Introduction . . . 443
23.2 Interpretation of the sample . . . 444
23.3 Sampling distributions . . . 448
23.4 Sampling distribution of $\bar{X}$ . . . 450
23.5 Confidence intervals . . . 454
23.6 Choice of sample size in a single experiment . . . 454
23.7 Choice of sample sizes for experiments with several treatments . . . 456
Problems . . . 458

Chapter 24 Parametric tests

24.1 Introduction . . . 465
24.2 One sample z-test . . . 465
24.3 One sample t-test . . . 470
24.4 Two sample z-test . . . 474
24.5 Two sample t-test . . . 475
24.6 Paired comparisons . . . 478
24.7 Transforming data . . . 480
Problems . . . 483

Chapter 25 Non-parametric tests

25.1 Introduction . . . 487
25.2 One sample binomial or sign test . . . 487
25.3 Two sample Mann-Whitney test . . . 489
25.4 Wilcoxon paired comparison test . . . 493
25.5 Kruskal-Wallis test for several treatments . . . 496
25.6 Friedman test . . . 499
25.7 Testing for inter-relationships, using contingency tables . . . 503
25.8 Non-parametric and parametric tests . . . 509
Problems . . . 510

Appendix A Trigonometric identities . . . 517
Appendix B Standard derivatives and integrals . . . 519
Appendix C Values of natural logarithms . . . 520
Appendix D Values of exponential functions . . . 522
Appendix E Areas under standard normal curve, $\Phi(z)$ . . . 524
Appendix F Upper tail critical values for Student's t-distribution . . . 525
Appendix G Lower tail critical values for Mann-Whitney test . . . 526
Appendix H Lower tail critical values for Wilcoxon signed ranks test . . . 527
Appendix I Critical values for χ^2-distribution . . . 528

Answers to Problems . . . 529

References . . . 571

Index . . . 573