Contents

1	**Infinite-dimensional Linear Programs**	1
	1.1 Introduction	1
	1.2 Some infinite linear programs	2
	1.2.1 The bottleneck problem	2
	1.2.2 Continuous-time network flow	3
	1.2.3 Cutting and filling	5
	1.2.4 Approximation	7
	1.2.5 Games	7
	1.3 Elements of the simplex method	9
	1.3.1 Duality	9
	1.3.2 Basic solutions and pivoting	12
	Problems	15
2	**Algebraic Fundamentals**	16
	2.1 Introduction	16
	2.2 Linear programs and their algebraic duals	16
	2.2.1 The linear program	17
	2.2.2 The algebraic dual problem	18
	2.3 Basic solutions	19
	2.3.1 Degeneracy	21
	2.3.2 Minimal support	21
	2.3.3 Existence of a basic optimal solution	23
	2.4 Reduced costs	24
	2.5 The pivot step	26
	2.6 Algebraic duality theory	28
	2.7 Notes	31
	Problems	32
3	**Topology and Duality**	35
	3.1 Introduction	35
	3.2 Dual pairs of topological vector spaces	36
	3.3 The linear programming problem	38
	3.3.1 Elementary duality theory	38

3.4 Subconsistency and subvalues 40
3.4.1 An example of Gale 42
3.4.2 An example of Kretschmer 43
3.5 Classification schemes 45
3.5.1 An example of Ben-Israel, Charnes and Kortanek 47
3.5.2 Homogenous constraints 48
3.6 Conditions for the absence of a duality gap 51
3.6.1 Closedness conditions 52
3.6.2 Interior point conditions 54
3.6.3 Boundedness conditions 56
3.6.4 Compactness conditions 57
3.7 The existence of optimal solutions 59
3.8 Notes 61
Problems 63

4 Semi-infinite Linear Programs 65
4.1 Introduction 65
4.2 The countable semi-infinite program and its dual 65
4.2.1 An example of Karney 69
4.3 Simplex extensions for countable semi-infinite programs 70
4.3.1 An algorithm for the dual problem 70
4.3.2 An algorithm for the primal problem 72
4.4 The continuous semi-infinite program and its dual 73
4.5 Simplex extensions for continuous semi-infinite programs 77
4.5.1 An algorithm for the dual problem 77
4.5.2 An algorithm for the primal problem 78
4.5.3 An example to illustrate the primal and dual algorithms 83
4.6 The uniform approximation problem 84
4.7 Notes 88
Problems 89

5 The Mass-transfer Problem 91
5.1 Introduction 91
5.2 The continuous transportation problem 91
5.2.1 The dual problem 92
5.2.2 Solvability and duality 93
5.3 Basic solutions and piecewise continuous assignments 95
5.4 An algorithm for the continuous transportation problem 99
5.4.1 Convergence of the algorithm 103
5.5 The semi-infinite transportation problem 106
5.6 Notes 109
Problems 110

6 Maximal Flow in a Dynamic Network 111
6.1 Introduction 111
6.2 The time-continuous network-flow problem 112
6.3 Generalized cuts 115
6.4 Reachable nodes 117
6.5 Duality theory 123
6.6 Notes 128
Problems 128

7 Continuous Linear Programs 130
7.1 Introduction 130
7.2 The continuous linear program and its dual 130
7.3 Separated continuous linear programs (SCLP) 135
7.4 The basic solutions for SCLP 136
7.5 A class of SCLPs with piecewise linear solutions 139
7.6 Duality theory 142
7.7 Notes 144
Problems 145

8 Other Infinite Linear Programs 146
8.1 Introduction 146
8.2 The capacity problem 147
8.2.1 Notes 150
8.3 The continuous network program 150
8.3.1 Notes 152
8.4 The space-continuous maximum-flow problem 152
8.4.1 Notes 156
8.5 Optimal design of structures 156
8.5.1 Notes 159

Index of Problem Statements 168

Subject Index 169