

CONTENTS

CHAPTER I - <u>DIFFERENTIAL CALCULUS IN NORMED LINEAR SPACES</u>		
1. Gateaux derivatives	...	1
2. Taylor's formula	...	7
3. Convexity and Gateaux differentiability	...	11
4. Gateaux differentiability and weak lower semi-continuity		14
5. Commutation of derivations	...	
6. Frechet derivatives	...	6
7. Model problem	...	19
CHAPTER II - <u>MINIMIZATION OF FUNCTIONALS-THEORY</u>		
1. Minimization without convexity conditions	...	21
2. Minimization with convexity conditions	...	25
3. Applications to the model problem and reduction to variational inequality	...	30
4. Some functional spaces	...	33
5. Examples	...	36
CHAPTER III - <u>MINIMIZATION WITHOUT CONSTRAINTS - ALGORITHMS</u>		
1. Method of descent :(1.1) Generalities	...	51
	(1.2) Convergent choices of the direction of descent w_k ...	54
	(1.3) Convergent choices of ρ_k	59
	(1.4) Convergence of algorithms	69
2. Generalized Newton's method	...	73
3. Other methods	...	87

CHAPTER IV - <u>MINIMIZATION WITH CONSTRAINTS- ALGORITHMS</u> ...	88
1. Linearization method ...	88
2. Centre method ...	105
3. Method of gradient with projection ...	109
4. Minimization in a product space ...	113
(4.1) Statement of the problem ...	113
(4.2) Minimization with constraints of convex functionals on products of reflexive Banach spaces ...	116
(4.3) Main result-convergence of the algorithm and Gauss-Seidel method ...	118
(4.4) Some applications-differentiable and non- differentiable functionals in finite dimensions	124
(4.5) Minimization of quadratic functionals on Hilbert spaces - Relaxation method by blocks ...	126
(4.6) Algorithm (of relaxation method)-Details ...	128
(4.7) Convergence of the algorithm ...	130
(4.8) Some examples of relaxation method in finite dimensional spaces ...	139
(4.9) Examples in infinite dimensional Hilbert spaces- optimization with constraints in Sobolev spaces	142
CHAPTER V - <u>DUALITY AND ITS APPLICATIONS</u> ...	144
1. Preliminaries - Recollection of Hahn-Banach and Ky Fan and Sion theorem, Lagrangian and Lagrange multipliers, Primal and dual problems ...	145
2. Duality in finite dimensional spaces via Hahn-Banach theorem ...	155
Qualifying hypothesis ...	159
Some examples of qualifying hypothesis ...	160

3.	Duality in infinite dimensional spaces via Ky Fan and Sion theorem	...	162
	(3.1) Duality in the case of a quadratic form	...	163
	(3.2) Dual problem	...	170
	(3.3) Method of Uzawa	...	173
4.	Minimization of non-differentiable functionals using duality - examples and algorithm	...	183
VI -	<u>ELEMENTS OF THEORY OF OPTIMAL CONTROL AND ELEMENTS OF OPTIMAL DESIGN</u>	...	194
1.	Optimal control theory	...	194
	(1.1) Formulation of the problem of optimal control		196
	(1.2) Duality and existence	...	198
	(1.3) Elimination of state	...	207
	(1.4) Approximation	...	211
2.	Theory of optimal design	...	215
	(2.1) Formulation of the problem of optimal design		217
	(2.2) A simple example	...	221
	(2.3) Computation of the derivative of the cost function		222
	(2.4) Hypothesis and results	...	226
	BIBLIOGRAPHY	...	233