CONTENTS

1	INTRODUCTION	1
1.1	Operations Research	1
1.2	Nature and Scope of Operations Research History and Development of Operations Research Overview of Operations Research Methods Perspective of This Text Organization of This Text	2 3 4 5 6
1.3		
1.4		
1.5		
1.6		
	Selected References	6
	Exercises	6
PAR	RT 1	
LIN	EAR PROGRAMMING	9
2	LINEAR PROGRAMMING MODELS AND APPLICATIONS	11
2.1	Formulating Linear Programming Models	11
	An Introductory Resource Allocation Problem	11
	Assumption of Continuity	13
	Sensitivity of the Optimal Solution	14
	Algebraic Statement of Linear Programming Problems	15
2.2	Further Linear Programming Formulation Examples	15
	Brewery Problem	16
	Oil Refinery Problem	18
	Warehouse Problem	19
	Chicken and the Egg Problem	21
	Nurse Scheduling Problem	23
2.3	Some Scientific Applications of Linear Programming	24
	Curve Fitting	24
	Inconsistent Systems of Equations	26
	Feasibility Problem	28

	Selected References Exercises	29 30
3	THE SIMPLEX ALGORITHM FOR LINEAR PROGRAMMING	37
3.1	Standard Form and Pivoting	37
	Standard Form	37
	The Simplex Tableau	40
	Pivoting on a Simplex Tableau	41
3.2	Canonical Form	42
	Canonical Form	42
	Finding a Better Basic Feasible Solution	44
	The Simplex Rule for Pivoting	45
	The Geometry of a Pivot	46
3.3	Optimal, Unbounded, and Infeasible Forms	47
	Optimal Form	47
	Unbounded Form	48
	Two Infeasible Forms	49
3.4	Solving Linear Programs in Canonical Form	51
	Pivoting to Optimal Form	51
	What Can Go Wrong: Degeneracy and Cycling	52 54
	Ways to Prevent Cycling	55 55
	Convergence in the Nondegenerate Case Convergence in the Degenerate Case	55 55
3.5	Obtaining Canonical Form from Standard Form	58
3.3	Getting an Identity	58
	The Subproblem Technique	59
	Pivoting to Form a Subproblem	62
	Summary of the Subproblem Technique	62
3.6	The Simplex Algorithm	63
	The Simplex Algorithm	63
3.7	Reformulating Any Linear Program into Standard Form	64
	Maximization Problems	64
	Inequality Constraints	64
	Free Variables	65
3.8	The Method of Artificial Variables	69
	Getting $\mathbf{b} \geq 0$	69
	The Artificial Problem	70
	Feasibility of the Original Problem	71
	Canonical Form for the Original Problem	71
3.9	Pivot Matrices and the Revised Simplex Method	74
	Pivot Matrices	74
	The Revised Simplex Method	76
	Tableaus with the Same Basic Sequence	78
3.10	Computer Solution of Linear Programs	79
	Computer Cycling	79
	Controlling Roundoff Errors	80
	Tolerances and Errors Other Pression Considerations	81 82
	Other Practical Considerations	
	Selected References Exercises	83 83
	P. LPHCISPS	^ ^

4	GEOMETRY OF THE SIMPLEX ALGORITHM	91
4.1	Geometry of Pivoting	91
	Graphical Representation of the Feasible Set	91
	Extreme Points and Basic Feasible Solutions	93
	The General Case	96
	Alternative Representation of the Feasible Set	97
4.2	Graphical Interpretation of Canonical Form Tableaus	98
4.2	Convex Sets	99
	Definition of a Convex Set Convexity of the Feasible Set	99 100
4.3	Multiple Optimal Solutions	
7.0	Finding All Optimal Solutions	101 101
	Convexity of the Set of Optimal Solutions	102
	Optimal Rays	103
	Selected References	105
	Exercises	105
5	DUALITY IN LINEAR PROGRAMMING	109
5.1	The Standard Dual Pair	109
	Duality Relations	110
5.2	Getting the Dual Solution from a Primal Solution	111
_	Relationships between Optimal Tableaus	11
	Constructing an Optimal Dual Vector	113
5.3	Economic Interpretation of Dual Variables	115
	The Complementary Slackness Conditions	118
5.4	Finding the Dual of Any Linear Program	119
	Dual of a Linear Program in Standard Form	120
	A More Complicated Example	123
	Dual of the Transportation Problem	122
5.5	The Dual Simplex Method	124
	Another Example of Dual Simplex Pivoting	127
5.6	Using the Dual to Computational Advantage	128
	A Difficult Primal Problem Having an Easy Dual	128
5.7	Theorems of the Alternative	130
	Selected References	132
	Exercises	133
6	SENSITIVITY ANALYSIS IN LINEAR	139
	PROGRAMMING	
6.1	The Brewery Problem	139
6.2	Changes in Production Requirements	141
	Changes in Nonbasic Variables	14
	Increasing Basic Variables	142
	Decreasing Basic Variables	144
	When a Nonbasic Variable Exceeds Its Minimum Ratio	14:
6.3	Changes in Available Resources	14:
	Changes in One Resource	140
	Simultaneous Changes in Several Resources	148

xiv CONTENTS

6.4	Changes in Selling Prices	151
	Simultaneous Changes in Prices	153
6.5	Adding New Products or Constraints	154
	New Products	154
	Technology Changes	155
	New Constraints	156
	Selected References	157 157
	Exercises	137
7	LINEAR PROGRAMMING MODELS FOR NETWORK FLOW PROBLEMS	161
7.1	The Transportation Problem	161
	The Transportation Tableau	166
7.2	Using the Dual to Improve the Current Solution	167
	Obtaining an Optimal Transportation Tableau	174
	A Final Comparison with the Simplex Algorithm	175
7.3	The Transportation Algorithm	176
7.4	Finding an Initial Basic Feasible Solution	178
7.5	Variations of the Transportation Problem	181
	Unequal Supply and Demand	181
	_ The Transshipment Problem Multiple Optimal Solutions	183 186
7 (• •	187
7.6 7.7	The Assignment Problem General Network Models	190
/./	The General Network Flow Model	191
	Spanning Trees and Basic Feasible Solutions	193
	Solving a General Network Flow Problem	195
	Summary of the General Network Flow Algorithm	198
	Finding an Initial Feasible Spanning Tree	199
	Selected References	201
	Exercises	201
	T II EGER, NONLINEAR AND DYNAMIC	
	GRAMMING	209
8	INTEGER PROGRAMMING	211
8.1	The Integer Programming Problem	211
8.2	Implicit Enumeration	214
8.3	Solution by Branch and Bound	217
312	The Branch-and-Bound Algorithm	217
	A Branch-and-Bound Example in Detail	218
	The Order of Selecting Unfathomed Nodes	225
	Practical Considerations in Using Branch and Bound	227
8.4	0-1 Integer Programs	227
	A 0–1 Example	228 233
	Looking Ahead	233 237
	How Far to Look Ahead Getting Nonnegative, Increasing Cost Coefficients	238
	The Branch-and-Bound Algorithm for 0–1 Programs	239

8.5	Integer Programming Formulation Examples	242
	The Oakwood Furniture Problem Revisited	242
	The Knapsack Problem	243
	Capital Budgeting	243
	Facility Location	244
	The Traveling Salesman Problem	246
	A Scheduling Problem	247
8.6	Integer Programming Formulation Techniques	250
	Writing General Integer Programs as 0–1 Programs	250
	Mixed-Integer Programs	251
	Enforcing Logical Conditions	251
	Selected References	253
	Exercises	253
9	NONLINEAR PROGRAMMING	259
9.1	A Nonlinear Programming Problem	259
	Contour Plots	261
	Assumption of Continuity	263
	Algorithms for Nonlinear Programming	263
9.2	Unconstrained Minimization	264
	Maxima and Minima in One Dimension	264
	Maxima and Minima in n Dimensions	267
9.3	Equality Constraints	272
	Parametric Representation of Equality Constraints	273
	Several Equality Constraints	278
	Several Parameters	280
	The Method of Lagrange	280 283
	Classifying Lagrange Points An Important Use of the Lagrange Multiplier Theorem	286
0.4		287
9.4	Inequality Constraints	288
	Active and Inactive Constraints The Orthogonality Condition	290
	The Karush–Kuhn–Tucker Conditions	291
9.5		295
9.3	Convexity Convex Functions	295
	Convexity and Minima	296
	Checking Whether a Function is Convex	297
9.6	Karush-Kuhn-Tucker Theory of Nonlinear Programming	299
7.0	The KKT Method	300
	Strategy in Solving the KKT Conditions	302
9.7	Numerical Methods of Nonlinear Programming	304
7• •	Line Searching	306
	The Method of Steepest Descent	307
	The Generalized Reduced-Gradient Method	311
	The Ellipsoid Algorithm	315
	Conclusion	322
9.8	Nonlinear Programs with Special Structure	322
	Quadratic Programming	322
	Geometric Programming	326
	Separable Programming	329
	Other Theoretical Aspects of Nonlinear Programming	332

xvi CONTENTS

	Selected References Exercises	333 334
10	DYNAMIC PROGRAMMING	347
10.1	An Introductory Example	347 347
	The Stagecoach Problem	347
	The Backward Recursive Relations	349
	The Forward Recursive Relations	351
	Basic Features of a Dynamic Programming Formulation	352
10.2	A Loading Problem	353
10.3	The Boxes Problem	356
10.4	An Equipment Replacement Problem	359
10.5	Problems with Several State Variables	363 364
10 (The Curse of Dimensionality	
10.6	Continuous State Dynamic Programming Problems	364
	A Simple Example A More Complicated Example	364 366
	Selected References Exercises	369 369
	DACT CLOCK	307
	T III BABILISTIC MODELS	375
11	QUEUEING MODELS	377
11.1	A Simple Example and Basic Terminology	377
11.2	A Simple Model Suggested by Observation	379
	Consequences of Exponentially Distributed Interarrival Times	382 387
	Special Case 1: Arrivals But No Departures Special Case 2: Departures But No Arrivals	390
11.3	A More General Arrivals and Departures Model	391
11.4	Steady-State Queueing Systems	394
11.5	A Single-Server System with Constant Rates	397
11.6	Operating Characteristics from Basic Principles	399
11.7	Multiple-Server Queues	402
11.8	Finite Queues	405
11.9	Limited-Source Queues	407
11.10	Optimization in Queues	409
	Optimizing over the Number of Servers, s	411
	Optimizing over the Mean Service Rate, µ	412 412
11.11	Optimizing over the Mean Arrival Rate, \(\lambda \)	
11.11	Queueing Models Involving Nonexponential Distributions The Kendall Notation	413 414
	Selected References Exercises	415 415
12	INVENTORY MODELS	421
12.1	Economic Order-Quantity Models	421
	Economic Order-Quantity Models with No Shortages Allowed	422
	Economic Order-Quantity Models with Shortages Allowed	424
	Economic Order-Quantity Models with Price Breaks	426
	Economic Order-Quantity Models with Several Inventories	427

12.2	Dynamic Inventory Models with Periodic Review	429
	Concave Cost Functions	432
	Convex Cost Functions	437
12.3	Stochastic Inventory Models	441
	A Single-Period Model	441
	A Two-Period Model	449
	Selected References	452
	Exercises	452
13	SIMULATION	457
13.1	Next-Event Simulation	458
	Observing a Queueing System	459
	Constructing an Event List	459
13.2	Statistical Analysis of Simulation Results	463
	Estimating L and L_q	463
	Estimating W and W_q	464
	Estimating Server Utilization	464
	Estimating Probability Distributions	464
40.0	Checking Random Interevent Times	465
13.3	Generating Pseudorandom Numbers	466
	Random and Pseudorandom Number Sequences	467
	Random-Number Generators	468
	The Multiplicative Congruential Algorithm	469
	Other Uniform Random-Number Generators Nonuniform Distributions	471
13.4	Monte Carlo Simulations	473
10.7	Evaluation of an Integral	475
	The Metropolis Algorithm	475 477
13.5	Practical Considerations in Simulation Modeling	480
1010	Computer Programs for Simulation	
	Logical and Statistical Validity	480 484
	Selected References	487
	Exercises	487
	A DDEAUNIS/	
	APPENDIX NOTATIONAL CONVENIENCE FOR MATTERIAL	491
	NOTATIONAL CONVENTIONS FOR MATRIX ALGEBRA	
A.1	Matrices	491
	Equality of Two Matrices	492
	Multiplying Two Matrices	492
A.2	Systems of Linear Equations	493
A.3	The Sum of Two Matrices	494
A.4	Multiplying a Matrix by a Real Number	494
A.5	The Transpose of a Matrix	494
A.6	Some Simple Matrix Identities	495
	Selected References	495
	Answers to Selected Exercises	493 497
	Index	505
		202