

Contents

1	Matrix Eigenvalue Methods	1
1.1	Introduction	1
1.2	Power Method for Diagonalization	5
1.3	The Rayleigh Quotient Gradient Flow	14
1.4	The QR Algorithm	29
1.5	Singular Value Decomposition (SVD)	34
1.6	Standard Least Squares Gradient Flows	36
2	Double Bracket Isospectral Flows	45
2.1	Double Bracket Flows for Diagonalization	45
2.2	Toda Flows and the Riccati Equation	59
2.3	Recursive Lie-Bracket Based Diagonalization	69
3	Singular Value Decomposition	85
3.1	SVD via Double Bracket Flows	85
3.2	A Gradient Flow Approach to SVD	88
4	Linear Programming	107
4.1	The Role of Double Bracket Flows	107
4.2	Interior Point Flows on a Polytope	116
4.3	Recursive Linear Programming/Sorting	122

5	Approximation and Control	133
5.1	Approximations by Lower Rank Matrices	133
5.2	The Polar Decomposition	150
5.3	Output Feedback Control	152
6	Balanced Matrix Factorizations	169
6.1	Introduction	169
6.2	Kempf-Ness Theorem	171
6.3	Global Analysis of Cost Functions	173
6.4	Flows for Balancing Transformations	177
6.5	Flows on the Factors X and Y	190
6.6	Recursive Balancing Matrix Factorizations	196
7	Invariant Theory and System Balancing	205
7.1	Introduction	205
7.2	Plurisubharmonic Functions	208
7.3	The Azad-Loeb Theorem	210
7.4	Application to Balancing	212
7.5	Euclidean Norm Balancing	223
8	Balancing via Gradient Flows	231
8.1	Introduction	231
8.2	Flows on Positive Definite Matrices	233
8.3	Flows for Balancing Transformations	246
8.4	Balancing via Isodynamical Flows	249
8.5	Euclidean Norm Optimal Realizations	257
9	Sensitivity Optimization	267
9.1	A Sensitivity Minimizing Gradient Flow	267

9.2	Related L^2 -Sensitivity Minimization Flows	279
9.3	Recursive L^2 -Sensitivity Balancing	287
9.4	L^2 -Sensitivity Model Reduction	291
9.5	Sensitivity Minimization with Constraints	293
A	Linear Algebra	307
A.1	Matrices and Vectors	307
A.2	Addition and Multiplication of Matrices	308
A.3	Determinant and Rank of a Matrix	308
A.4	Range Space, Kernel and Inverses	309
A.5	Powers, Polynomials, Exponentials and Logarithms	310
A.6	Eigenvalues, Eigenvectors and Trace	310
A.7	Similar Matrices	311
A.8	Positive Definite Matrices and Matrix Decompositions	312
A.9	Norms of Vectors and Matrices	313
A.10	Kronecker Product and Vec	314
A.11	Differentiation and Integration	314
A.12	Lemma of Lyapunov	315
A.13	Vector Spaces and Subspaces	315
A.14	Basis and Dimension	316
A.15	Mappings and Linear Mappings	316
A.16	Inner Products	317
B	Dynamical Systems	319
B.1	Linear Dynamical Systems	319
B.2	Linear Dynamical System Matrix Equations	320
B.3	Controllability and Stabilizability	321
B.4	Observability and Detectability	322

B.5	Minimality	323
B.6	Markov Parameters and Hankel Matrix	323
B.7	Balanced Realizations	323
B.8	Vector Fields and Flows	324
B.9	Stability Concepts	326
B.10	Lyapunov Stability	327
C	Global Analysis	331
C.1	Point Set Topology	331
C.2	Advanced Calculus	334
C.3	Smooth Manifolds	337
C.4	Spheres, Projective Spaces and Grassmannians	339
C.5	Tangent Spaces and Tangent Maps	342
C.6	Submanifolds	345
C.7	Groups, Lie Groups and Lie Algebras	347
C.8	Homogeneous Spaces	350
C.9	Tangent Bundle	353
C.10	Riemannian Metrics and Gradient Flows	355
C.11	Stable Manifolds	357
C.12	Convergence of Gradient Flows	360
	Bibliography	363
	Author Index	379
	Subject Index	383