

Contents

PREFACE	xi
INTRODUCTION	xiii
PART I FOUNDATIONS OF THEORY AND METHODS OF FINITE-DIMENSIONAL OPTIMIZATION	1
1. Basic Concepts, Problems and Fields of Applications of Perturbation Theory	3
1.1. Basic Concepts of Perturbation Theory	3
1.1.1. Concepts of stability in a mathematical programming problem (MPP)	3
1.1.2. Perturbation of MPP	5
1.1.3. Classification of perturbations	6
1.1.4. Definitions of MPP stability with respect to perturbation	7
1.2. Basic Problems of Perturbation Theory. Counterexamples	9
1.3. Brief Review of Investigations of Perturbation Theory	18
1.4. Vector Optimization and Parametric Extremum Problems	23
1.5. Accuracy Estimate of Optimization Problem Solutions. Models of Decision Making under Uncertainty Conditions	25
1.6. Two-level Systems and Parametric Optimization	28
1.6.1. Two-level optimization	29
1.6.2. Economical equilibrium models (noncooperative)	30
1.6.3. Inter-section cooperation models	31
1.7. Approximation by a Sequence of Extremum Problems and Convergence of Numerical Methods	32
1.8. Dynamic Programming and the Problem of the Differentiability of the Bellman Function	33
1.9. Mathematical Programming Problems with a Block Structure and Related Perturbations	34
1.9.1. Parametrization and perturbation function in a block problem with binding variables	35
1.9.2. Parametrization and perturbation function in a block problem with binding constraints	37
1.9.3. Parametrization in the case of both binding constraints and binding variables	39
1.10. Nonsmooth Optimization and Perturbation Functions	41
1.11. Asymptotic Method of Constructing an Approximate Solution for Optimization Problems with Small Parameters	45
1.12. Correction of Solutions of Optimization Problems With Small Variations of Original Data	51

2. Perturbation Theory of Smooth Mathematical Programming	
Problems (Main Results)	53
2.1. Theorem on Estimate of Distance to the Set $Q(w)$	53
2.2. Zero Approximation Theory	54
2.2.1. Properties of optimal solutions to the generating problem	54
2.2.2. Stability with respect to optimal value and solution set	57
2.2.3. Lipschitz property of the perturbation function	59
2.2.4. Stability of the set of optimal Lagrange's multipliers. Local stability of an active index	59
2.2.5. On one property of approximate solutions to the perturbed problem	59
2.3. First Approximation Theory	60
2.3.1. First-approximation problem for the perturbed problem and its properties	60
2.3.2. First-order differential expansions of the perturbation function	62
2.3.3. Differential properties of approximate, $o(\varepsilon)$, solutions to the perturbed problem	65
2.4. Second Approximation Theory	66
2.4.1. Second-approximation problem for the perturbed problem and its properties	66
2.4.2. Second-order differential expansions of the perturbation function	70
2.4.3. Differential properties of approximate, $o(\varepsilon^2)$, solutions to the perturbed problem	71
3. Examples of Studies in Parametric Optimization Problems.	
Applications of Perturbation Theory	73
3.1. Dependence of the Optimal Value and the Solutions to a Linear Programming Problem on All of its Coefficients	73
3.2. Investigation of Two Classes of Parametric Convex Programming Problems	77
3.2.1. Dependence of optimal value and solutions on the weight coefficients in a criterion and on the right-hand side of constraints	77
3.2.2. Dependence of the projection of the point ξ onto the convex set $Q(\eta)$ on the parameters $w = (\xi, \eta)$	80
3.3. An Illustrative Example: Analysis of the Perturbation of a Two-Dimensional Problem with Quadratic Separable Functions	83
3.3.1. Properties of the generating problem	83
3.3.2. Stability	84
3.3.3. First-approximation theory	84
3.3.4. Second-approximation theory	87
3.4. A Small Perturbation of the Cauchy–Bunyakovsky Inequality by Quadratic Forms	93
3.5. Decomposition Method of Choice of Descent Direction in a Block Problem with Binding Variables	95
3.6. Asymptotic Method of Constructing the Approximate Solution for the Perturbation of a Block Problem with Binding Variables and Binding Constraints	99
3.7. Asymptotic Method of Constructing the Approximate Solution for the Perturbation of a Problem that Can Be Aggregated with Respect to a Group of Variables	106

PART II BASIC PERTURBATION THEORY IN FINITE-DIMENSIONAL OPTIMIZATION **109**

4. Elements of Convex and Nonconvex Analysis	111
4.1. Convex Sets and Convex Functions	111

4.1.1. Algebraic properties of convex sets	111
4.1.2. Topological properties of convex sets	113
4.1.3. Convex and quasiconvex, uniformly convex and quasiconvex, strongly convex functions and their properties	113
4.1.4. Properties of quasiconvex function minima on a convex set	122
4.1.5. Projection operator onto a convex set	123
4.1.6. Separation theorems for convex sets	124
4.1.7. Properties of sublinear functions	125
4.1.8. Adjoint cones. Farkas' theorem	128
4.2. Differential Properties (Continuity, Lipschitz Property, Directional Differentiability) of Convex Functions	128
4.3. Definition and Properties of η -Subdifferentials at a Point, Supporting Vectors, and the Function Adjoint to a Convex One	139
4.4. Calculation of a Set of η -Subdifferentials at a Point, a Set of Supporting Vectors, and a Function Adjoint to a Convex One	147
4.5. Minimax Theorem. Criterion for Existence of a Saddle Point	156
4.6. Optimality Criteria in Convex Programming (Duality Theory, Optimality Conditions in Differential Form)	159
4.7. Compatibility Conditions and Estimate of Distance to Solution Set for a System of Convex Inequalities and Linear Equations. Hoffman's Theorem	168
4.7.1. Compatibility criterion for a system of convex inequalities and linear inhomogeneous equations	168
4.7.2. Incompatibility criteria for a system of strict convex inequalities, nonstrict affine inequalities and linear inhomogeneous equations	171
4.7.3. Estimate of distance to the solution set of a system of nonstrict convex inequalities and linear equations	174
4.7.4. Estimate of the distance to the solution set of a parametric system of linear inhomogeneous inequalities and equations (Hoffman's theorem)	175
4.8. Upper Convex, Fine Convex and Fine Linear Approximations of Functions Depending on Parameters	177
4.9. Locally Convex Functions. Classes $\mathcal{E}(\mathcal{Z}, z_0)$, $\mathcal{F}(\mathcal{Z}, z_0)$ and $\mathcal{G}(\mathcal{Z}, z_0)$ of Functions	182
4.10. Strict Differentiability with a Second-Order Remainder. Second-Order Approximation at a Point	193
4.11. Estimate of Distance to the Set of Solutions of a System of Convex Inequalities and Nonlinear Equations	194

5. Optimality Criteria in Generating Problem **201**

5.1. Definition of Necessary and Sufficient Optimality Conditions of First and Higher Orders	202
5.2. Necessary First-Order Minimum Condition	207
5.3. Sufficient First-Order Optimality Condition	213
5.4. Local Minimum Criteria for Nonconvex Problems	216
5.5. Criteria of Higher-Order Minimum	231
5.6. Necessary and Sufficient Second-Order Optimality Conditions	241
5.7. Examples of Studying an Extremum Problem for a Local Minimum	243
5.8. Local Minimum Criteria Without Regularity Condition	255

PART III PERTURBATION THEORY IN MATHEMATICAL PROGRAMMING	259
6. General Perturbation Theory	261
6.1. Estimate of Distance to the Set $Q(w)$	262
6.2. General Assumptions	265
6.3. Stability with Respect to Optimal Value and to Solution Set	267
6.4. Lipschitz Property of the Perturbation Function	270
6.5. Stability of the Set of Optimal Lagrange Multipliers. Local Stability of the Active Index $I_0 \in I(w_0, x_0)$. Properties of Approximate Solutions of the Perturbed Problem	271
6.6. First Approximation Problem	276
6.7. Main Results of First Approximation Theory	284
6.8. Proofs of Theorems of First Approximation Theory	290
6.9. Second Approximation Problem for the Perturbed Problem	301
6.10. Main Results of the Quadratic Theory	304
6.11. Proofs of Theorems of Quadratic Theory	306
6.12. General Theory for the Unbounded Set of Optimal Solutions to the Generating Problem	318
7. Perturbation Theory for Smooth Mathematical Programming Problems	323
7.1. First Approximation Theory for Problems with Functions \mathcal{J}, f_i , and g_j Continuously Differentiable with Respect to $\{w, x\}$	323
7.2. Second Approximation Theory for Problems with Functions \mathcal{J}, f_i and g_j Twice Differentiable with Respect to $\{w, x\}$	328
8. Perturbation Theory for Convex Programming Problems	329
8.1. Properties of Optimal Solutions to the Generating Problem and its Dual	329
8.2. Perturbation of a Convex Programming Problem and its Dual	331
8.3. Stability with Respect to Optimal Value and Solution Set in the Perturbed and Dual Problems	335
8.4. First and Second Approximation Theory for the Perturbed Problem	338
8.5. First Approximation Theory for the Dual Perturbed Problem. Differential Expansion for the Dual Gap	343
9. Theory of Minimax Perturbations Under Bound Constraints	349
9.1. Problem Posing, Notation and Assumptions	349
9.2. The Zero, First and Second Approximation Theories	354
NOTES AND BIBLIOGRAPHICAL COMMENTS	359
REFERENCES	367
INDEX	381