

## TABLE OF CONTENTS

Conditions for Global Optimality <i>J.-B. Hiriart-Urruty</i>	1
Complexity Issues in Global Optimization: A Survey <i>Stephen A. Vavasis</i>	27
1. Introduction	27
2. Convex problems	27
3. Two nonconvex problems with efficient algorithms	30
4. General nonconvex problems	33
5. Approximation algorithms in quadratic programming	38
6. Conclusions	40
References	40
Concave Minimization: Theory, Applications and Algorithms <i>Harold P. Benson</i>	43
1. Introduction	43
2. Overview of Mathematical Properties	47
3. Some Key Properties of Concave Functions	49
3.1 Classical Properties	49
3.2 More Recent Properties	52
4. Basic Properties of Concave Minimization Problems	58
5. Overview of Applications	61
6. Direct Applications	62
6.1 Fixed Charge Problems	62
6.2 Multiplicative Problems	64
6.3 Min-Max Problems	66
6.4 Problems with Economics of Scale	66
7. Indirect Applications	67
7.1 Integer Programming Problems	68
7.2 Bilinear Programming Problems	69
7.3 D.C. Optimization Problems	70
7.4 Complementary Problems	73
7.5 Some Additional Indirect Applications	74
8. Overview of Three Fundamental Algorithmic Approaches	77
9. Enumerative Methods	79
9.1 Extreme Point Ranking	79
9.2 Pure Cutting Plane Approaches	81
9.3 Combinations of Cuts	83
9.4 Cone Covering Approaches	88
10. Successive Approximation Methods	91
10.1 Outer Approximation	92
10.2 Polyhedral Annexation	99
10.3 Successive Underestimation	103
11. Successive Partitioning Methods	106
11.1 A Prototype Successive Partitioning Algorithm	107

11.2 Analysis of the Prototype Successive Partitioning Algorithm	109
11.3 Simplicial Successive Partitioning Algorithms	110
11.4 Rectangular Successive Partitioning Algorithms	117
11.5 Conical Successive Partitioning Algorithms	123
12. Some Future Research Needs	134
Acknowledgements	137
References	137
<b>D.C. Optimization: Theory, Methods and Algorithms</b>	
<i>Hoang Tuy</i>	149
1. Introduction	149
2. Some Typical Examples	151
3. D.C. Functions	156
4. D.C. Sets	160
5. Global Optimality Criterion	163
6. Duality in Global Optimization	167
7. Primal Methods in Reverse Convex Programming	172
7.1 Generalities	173
7.2 Outer Approximation Method	174
7.3 Cut and Split (Branch and Bound) Methods	176
7.4 Combined Methods	179
8. Dual Methods in Reverse Convex Programming	181
8.1 Polyhedral Annexation Method	181
9. Noncanonical D.C. Optimization Problems	184
9.1 Problems with Separated Nonconvexity	184
9.1.1 Simplicial Algorithm	184
9.1.2 Rectangular Algorithm	186
9.2 Problems with General Convex Constraints	188
9.3 Problems with General Nonconvex Objective Function	190
9.3.1 Biconvex and Convex-Concave Programming	190
9.3.2 Factorable Programming	192
10. Continuous Optimization Problems	194
10.1 The Relief Indicator Method	194
10.2 Outer Approximation Algorithm	197
10.3 Combined OA/BB Algorithm	199
10.4 Partitioning Method	200
11. Special Methods for Special Problems	201
References	209
<b>Quadratic Optimization</b>	
<i>Christodoulou A. Floudas and V. Visweswaran</i>	217
1. Introduction	217
2. The General Quadratic Programming Problem	218
2.1 Classification of Quadratic Programming Problems	219
3. Optimality Conditions and Solution Characterization	220
3.1 Feasible Descent Directions and Optimality Conditions	220
3.2 Active Constraints and Optimality	221

3.3 Global Optimality Criteria	222
4. Complexity Issues	222
5. Bilinear Programming Problems	224
5.1 Transformation of Quadratic Problems to Bilinear Problems	228
6. Concave Problems	228
6.1 Extreme Point Ranking Methods	228
6.2 Cutting Plane Methods	229
6.3 Convex Envelopes	230
6.4 Reduction to Bilinear Programming	231
6.5 Solution of Large Concave QPs with Linear Terms	232
6.6 Reduction to Separable Form	234
7. Indefinite Quadratic Problems	235
7.1 Quadratic Problems with Box Constraints	235
7.2 Decomposition Techniques	239
7.3 Large-Scale Indefinite Problems	241
7.4 Polynomial Time Algorithms	243
8. Quadratic Problems with Quadratic Constraints	244
8.1 Decomposition Techniques	247
9. Quadratic Optimization and the Linear Complementarity Problem	251
10. Applications	252
10.1 Economies of Scale	252
10.2 Pooling and Blending Problems	253
10.3 Multicomponent Separation Problems	254
10.4 The Quadratic Knapsack Problems	255
10.5 IC Circuit Layout and Compaction	256
10.6 Optimal Design of Trusses	257
10.7 Robust Stability Analysis of Feedback Control Systems	257
11. Summary of Computational Results	259
11.1 Bilinear Programs and Quadratically Constrained Problems	259
11.2 Concave Quadratic Problems	259
11.3 Indefinite Quadratic Problems	261
Conclusions	263
Acknowledgements	264
References	264
Complementary Problems	
<i>Jong-Shi Pang</i>	271
1. Introduction	271
2. Problem Definition	273
3. Source Problems	274
3.1 Mathematical Programs	275
3.2 Equilibrium Problems	277
3.3 Engineering Applications	280
4. Global Optimization Formulations	282
5. The KKT System of a VI	286
6. Existence Theory and Solution Properties	289
6.1 Degree Theory Concepts	289

6.2 Some Basic Results	289
6.3 Local Uniqueness of a Solution	293
6.4 The Affine VI	295
7. Iterative Algorithms	300
7.1 The Basic Newton Method	300
7.2 The Nonsmooth-Equations Approach	304
7.3 Interior Point Methods	308
8. Error Bounds	313
9. Sensitivity and Stability Analysis	317
9.1 A Local Stability Theory for Nonsmooth Equations	318
9.2 Local Sensitivity of VI with Fixed Set	320
9.3 Specialization to a Parametric KKT System	325
9.4 Parametric VI with Nonunique Multipliers	326
9.5 Global Sensitivity Analysis	327
10. Conclusions and Some Open Topics	328
Acknowledgement	329
References	329
<b>Minimax and Its Applications</b>	
<i>Ding-Zhu Du</i>	339
1. Introduction	339
2. Chebysherv Theorem	340
3. Linear Programming	341
4. Du-Hwang Theorem	342
5. Geometric Inequalities	345
6. Approximation Performance	348
7. Gilbert-Pollak Conjecture	350
8. Refinement of the Proof of Du and Hwang	351
8.1 Characteristic Area and Inner Spanning Trees	351
8.2 Critical Structure	357
8.3 Hexagonal Trees	361
9. Discussion	365
References	366
<b>Multiplicative Programming Problems</b>	
<i>Hiroshi Konno and Takahito Kuno</i>	369
1. Introduction	369
2. Linear Multiplicative Programming Problems	370
2.1 A Parametric Objective Simplex Algorithm	373
2.2 A Parametric Right-Hand-Side Simplex Algorithm	375
2.3 Other Methods for Linear Multiplicative Programs	377
3. Convex Multiplicative Programming Problems	378
3.1 Minimization of the Product of two Convex Functions	379
3.2 Minimization of the Product of Several Convex Functions	384
3.3 Minimization of the Product of Several Affine Functions	389
4. Other Problems Related to Multiplicative Functions	393
4.1 Minimization of a Sum of Convex Multiplicative Functions	393

4.2 Generalized Linear Multiplicative Program	396
4.3 Programs with a Convex Multiplicative Constraint	397
References	403
 Lipschitz Optimization	
<i>Pierre Hansen and Brigitte Jaumard</i>	407
1. Introduction	407
2. Univariate Lipschitz Optimization	410
2.1 Determination of the Globally Optimal Value	410
2.1.1 Finite Convergence	410
2.1.2 Tightest Upper Bounding Functions	411
2.1.3 Bounds on the Number of Evaluation Points	412
2.1.4 Piyavskii's Algorithm	413
2.1.5 Convergence of Piyavskii's Algorithm	415
2.1.6 Bounds on the Number of Iterations of Piyavskii's Algorithm	417
2.1.7 Number of Evaluation Points and Overestimation of the Lipschitz Constant	418
2.1.8 Evtushenko's Algorithm	421
2.1.9 Timonov's Algorithm and a Bayesian Analysis	422
2.1.10 Schoen's Algorithm Best $\epsilon$ -Optimal Strategies	424
2.1.11 Galperin's Algorithm in the Univariate Case	426
2.1.12 Hansen, Janmard and Lu's Two Phase Algorithm: Principle	427
2.1.13 Two-Phase Algorithm: Local Quasiconvexity or Quasiconcavity and Phase 1 Heuristic	428
2.1.14 Two-Phase Algorithm: Linear Approximation and Phase 2	431
2.1.15 Computational Comparison of Algorithms for Finding a Globally $\epsilon$ -Optimal Value	435
2.2 Locating One or All Globally Optimal Points	439
2.2.1 Basso's Revision of Piyavskii's Algorithm	439
2.2.2 An Algorithm for Locating All Globally Optimal Points	
2.2.3 Computational Results with the Algorithm for Locating All Globally Optimal Points	447
3. Unconstrained Multivariate Lipschitz Optimization: Exact Algorithms	448
3.1 Reduction to One-Dimensional Problems	449
3.1.1 Piyavskii's Nested Optimization Algorithm	449
3.2 Algorithms Using a Single Upper-Bounding Function	450
3.2.1 Piyavskii's and Mladineo's Algorithms	450
3.2.2 Mayne and Polak's Algorithm	452
3.2.3 Jaumard, Herrmann and Ribault's Algorithm	453
3.2.4 Wood's Algorithm	456
3.3 Branch-and-Bound Algorithms	459
3.3.1 General Framework	459
3.3.2 Galperin's Algorithms	461
3.3.3 Pointer's Algorithm	462
3.3.4 Meewella and Mayne's Algorithm	464
3.3.5 Gourdin, Hansen and Jeamard's Algorithm	466
3.4 Computational Comparison	467

4.	Unconstrained Lipschitz Optimization: Heuristic Algorithms	475
4.1	Unknown Lipschitz Constant: Strongin's Algorithm	475
4.2	Known Lipschitz Constant	478
5.	Constrained Lipschitz Optimization: Exact Algorithms	478
5.1	Extension of Branch-and-Bound Algorithms	478
5.2	Thach and Tuy's Algorithms	480
6.	Applications	483
6.1	Solution of Nonlinear Equations and Inequalities	483
6.2	Parametrization of Statistical Models	484
6.3	Calibration of Nonlinear Descriptive Models	484
6.4	Black Box System Optimization	484
6.5	Location Problems	484
6.6	Vehicular Traffic and Commuter's Departure Time	485
7.	Conclusions	485
	References	487

## Fractional Programming

*S. Schaible*

1.	Introduction	495
2.	Single-Ratio Fractional Programs	499
2.1	Applications	499
2.1.1	Economic Applications	499
2.1.2	Non-Economic Applications	502
2.1.3	Indirect Applications	502
2.2	Theoretical and Algorithmic Results	505
2.2.1	Direct Solution of the Quasiconcave Program (P)	505
2.2.2	Solution of an Equivalent Concave Program (P')	506
2.2.3	Solution of a Dual Program (D)	507
2.2.4	Solution of a Parametric Problem (Pq)	509
3.	Maximization of the Smallest of Several Ratio	511
3.1	Applications	511
3.2	Theoretical and Algorithmic Results	512
4.	Maximization of a Sum of Ratios	516
4.1	Applications	516
4.2	Theoretical and Algorithmic Results	517
5.	Multi-Objective Fractional Programs	519
6.	Conclusion	523
7.	Bibliography for Fractional Programming	524

## Network Problems

*G.M. Guisewite*

1.	Introduction	609
2.	Application areas	612
3.	Complexity of Network Problems	618
4.	Algorithms	622
4.1	Vertex Ranking Technique	623
4.2	Branch-and-Bound	623

4.3 Dynamic Programming	627
4.4 Decomposition Techniques	628
4.5 Local Search Algorithms	630
4.6 Specific Network Structure	634
4.7 Special Cost	637
4.8 Approximate Algorithms and Heuristics	639
4.9 Related Algorithms	641
5. Summary	641
References	642
 Trajectory Methods in Global Optimization	
<i>Immo Diener</i>	649
1. Introduction	649
2. Using Differential Equations	652
2.1 Griewank's Method	653
2.2 The Method of Snyman and Fatti	655
2.3 Branin's Method	655
3. Newton-Leaves	657
3.1 Geometry of Newton Leaves	657
3.2 The Graphs $\Gamma^k$	660
3.3 Connected Newton Leaves	661
4. Connection Trajectory Components	662
5. Final Remarks	665
References	666
 Homotopy Methods	
<i>W. Forster</i>	669
1. Introduction	669
2. List of Applications	670
2.1 Applications in Economics	670
2.2 Mathematical Applications	670
2.3 Engineering Applications	671
3. Historical Background of Piecewise Linear Algorithms	672
4. Fixed Point Theorems and Labelling Lemmas	673
5. Integer Labelling, Vector Labelling and Pivoting Tables	682
5.1 Pseudomanifolds and Labelling	682
5.1.1 Pseudomanifolds	682
5.1.2 Integer Labelling	683
5.1.3 Vector Labelling	684
5.1.4 Primitive Sets	686
5.2 Pivoting Tables	686
5.2.1 Pivoting Rules for the Triangulation K1	689
5.2.2 Pivoting Rules for the Triangulation J1	691
5.2.3 Pivoting Rules for the Triangulation D1	692
5.2.4 Triangulations with Continuous Refinement of Grid Size	696
6. Some Pivoting Algorithms on $\mathbb{R}^n$ (and $\mathbb{C}^n$ )	696
7. The Number of Homotopy Invariant Solutions	699

8. Kuhn's Algorithm for one Polynomial	704
9. Generalization to Systems of Polynomial Equations	709
9.1 Systems of n Polynomials in n Variables	709
9.2 Labelling Function	714
9.3 Estimates	715
9.4 Starting Procedure and Algorithm	716
9.5 Error Estimates	718
9.6 Computational Complexity	718
9.7 Systems of m Polynomials in n Variables	719
9.8 Systems with No Unique Dominating Term	720
10. Implications for Optimization Problems	721
10.1 Optimization Problems Considered as Problems in $\mathbb{R}^n$	721
10.2 Optimization Problems Considered as Problems in $\mathbb{C}^n$	722
11. Homotopy Methods Based on Differential Topology	724
11.1 Historical Background and Key Concepts	724
11.2 The Kellogg, Li, Yorke Algorithm and Extensions	726
12. Complementary Problems	731
13. Programming Issues	733
14. Further Reading	734
15. Concluding Remarks	735
16. References	737
Interval Methods	
<i>Helmut Ratschek and Jon Rokne</i>	751
Introduction	752
Part I: Interval Analysis	755
1.1 Advantages of Intervals	755
1.2 Interval Arithmetic Operation	757
1.3 Machine Interval Arithmetic	759
1.4 Further Notations	760
1.5 Inclusion Functions and Natural Interval Extensions	763
1.6 Centered Forms	765
1.7 Interval Newton Methods	770
1.8 The Hansen-Sengupta Version	772
Part II: Global Unconstrained Optimization	779
2.1 Introduction	779
2.2 The Basic Algorithm	781
2.3 Termination Criteria	785
2.4 Accelerating Devices	787
2.5 Bisections	791
Part III: Constrained Optimization	797
3.1 Introduction	797
3.2 Controlling the Feasible Domain	799
3.3 Trouble with Constraints	803
3.4 The Basic Algorithm	806

3.5 Inexactly Posed Problems	810
3.6 Convergence Conditions	811
3.7 Accelerating Devices	813
3.8 Numerical Examples	819
<b>Bibliography</b>	<b>824</b>
<b>Stochastic Methods</b>	
<i>C. Guus E. Boender and H. Edwin Romeijn</i>	829
1. Introduction	829
2. Two-Phase Methods	830
2.1 Pure Random Search	831
2.2 Multistart	832
2.3 Clustering Methods	832
2.3.1 Density Clustering	832
2.3.2 Single Linkage Clustering	833
2.4 Multi Level Single Linkage (MLSL)	834
3. Random Search Methods	835
3.1 Pure Random Search	835
3.2 Random Search	836
3.3 Pure Adaptive Search	837
3.4 Adaptive Search	838
4. Simulated Annealing	840
4.1 The Algorithm	840
4.2 Convergence	841
4.3 Examples	842
5. The Random Function Approach	846
5.1 Stochastic Processes	847
5.2 The One-Dimensional Wiener Process	850
5.3 The Multidimensional Case	851
6. Stopping Rules	853
6.1 Stopping Rules Based on the Local Optima Structure	856
6.1.1 Statistical Model	856
6.1.2 Non-Sequential Rules	857
6.1.3 Sequential Rules	858
6.1.4 Incorporation of the Function Values	859
6.2 Stopping Rules Based on the Distribution of Function Values	860
6.2.1 The Continuous Case	861
6.2.2 The Discrete Case	862
6.3 Conclusion	864
7. Concluding Remarks and Future Research	865
References	865
<b>Index</b>	<b>871</b>