

---

---

# Contents

<b>Introduction</b>	<b>xi</b>
<b>Chapter 1. Production Models: Maximizing Profits</b>	<b>1</b>
1.1 A two-variable linear program	2
1.2 The two-variable linear program in AMPL	5
1.3 A linear programming model	6
1.4 The linear programming model in AMPL	7
The basic model	8
An improved model	10
Error messages	12
1.5 Adding lower bounds to the model	13
1.6 Adding resource constraints to the model	14
<b>Chapter 2. Diets, Blending and Scheduling: Minimizing Costs</b>	<b>23</b>
2.1 A linear program for the diet problem	23
2.2 An AMPL model for the diet problem	26
2.3 Using the AMPL diet model	28
2.4 Generalizations for blending and scheduling	32
<b>Chapter 3. Transportation, Assignment and Minimum-Cost Flows</b>	<b>39</b>
3.1 A linear program for the transportation problem	40
3.2 An AMPL model for the transportation problem	41
3.3 Other interpretations of the transportation model	45
<b>Chapter 4. Building Larger Models</b>	<b>51</b>
4.1 A multicommodity transportation model	52
4.2 A multiperiod production model	55
4.3 A model of production and transportation	59
<b>Chapter 5. Simple Sets and Indexing</b>	<b>67</b>
5.1 Unordered sets	67
5.2 Sets of numbers	68

5.3	Set operations	70
5.4	Set membership operations and functions	72
5.5	Indexing expressions	73
5.6	Ordered sets	76
5.7	Syntax summary	80
<b>Chapter 6. Compound Sets and Indexing</b>		<b>83</b>
6.1	Sets of ordered pairs	83
6.2	Subsets and slices of ordered pairs	85
6.3	Sets of longer tuples	88
6.4	Operations on sets of tuples	90
6.5	Indexed collections of sets	92
6.6	Syntax summary	96
<b>Chapter 7. Parameters and Expressions</b>		<b>101</b>
7.1	Parameter declarations	102
7.2	Arithmetic expressions	103
7.3	Logical and conditional expressions	106
7.4	Restrictions on parameters	108
7.5	Computed parameters	110
7.6	Logical and symbolic parameters	112
7.7	Syntax summary	113
<b>Chapter 8. Linear Programs: Variables, Objectives and Constraints</b>		<b>119</b>
8.1	Variables	119
8.2	Linear expressions	123
8.3	Objectives	125
8.4	Constraints	129
8.5	Syntax summary	132
<b>Chapter 9. Specifying Data</b>		<b>135</b>
9.1	Set data	136
	One-dimensional sets	136
	Two-dimensional sets	137
	Higher-dimensional sets	139
9.2	Parameter data	140
	One-dimensional parameters	141
	Two-dimensional parameters	142
	Higher-dimensional parameters	145
	Default values	147
9.3	Variable data	148
9.4	Syntax summary	149
<b>Chapter 10. Command Environment</b>		<b>155</b>
10.1	General principles	156
	Commands	156
	Options	156

10.2	Setting up and solving models	158
	Entering models and data	158
	Solving a model	159
10.3	Browsing through results: the <code>display</code> command	161
	Displaying sets	162
	Displaying parameters and variables	163
	Displaying indexed expressions	166
	Output to a file	168
10.4	Formatting options for <code>display</code>	169
	Arrangement of lists and tables	169
	Control of line width	171
	Suppression of zeros	173
10.5	Numeric options for <code>display</code>	175
	Appearance of numeric values	175
	Rounding of solution values	179
10.6	Other output commands	181
	The <code>print</code> command	181
	The <code>printf</code> command	182
10.7	Related solution values	183
	Objective functions	183
	Bounds and slacks	184
	Dual values and reduced costs	186
10.8	Modifying and re-solving	188
	Changing the model	189
	Changing the data	190
	Re-solving after changes	192
10.9	Batch operation	193
<b>Chapter 11. Network Linear Programs</b>		<b>195</b>
11.1	Minimum-cost transshipment models	195
	A general transshipment model	196
	Specialized transshipment models	199
	Variations on transshipment models	202
11.2	Other network models	204
	Maximum flow models	204
	Shortest path models	205
	Transportation and assignment models	206
11.3	Declaring network models by <code>node</code> and <code>arc</code>	209
	A general transshipment model	210
	A specialized transshipment model	211
	Variations on transshipment models	212
	Maximum flow models	213
11.4	Rules for <code>node</code> and <code>arc</code> declarations	216
	<code>node</code> declarations	216
	<code>arc</code> declarations	216
	Interaction with objective declarations	217

Interaction with constraint declarations	218
Interaction with variable declarations	218
11.5 Solving network linear programs	219
<b>Chapter 12. Columnwise Formulations</b>	<b>229</b>
12.1 An input-output model	230
Formulation by constraints	230
A columnwise formulation	231
Refinements of the columnwise formulation	232
12.2 A scheduling model	234
12.3 General rules	238
<b>Chapter 13. Nonlinear Programs</b>	<b>241</b>
13.1 Sources of nonlinearity	242
Examples of nonlinear costs	242
Other sources of nonlinearity	245
13.2 Nonlinear variables	246
Initial values of variables	246
Automatic substitution of variables	247
13.3 Nonlinear expressions	248
13.4 Pitfalls of nonlinear programming	250
Function range violations	251
Multiple local optima	254
Other pitfalls	257
<b>Chapter 14. Piecewise-Linear Programs</b>	<b>265</b>
14.1 Cost terms	266
Fixed numbers of pieces	266
Varying numbers of pieces	268
14.2 Common two-piece and three-piece terms	269
Penalty terms for “soft” constraints	269
Dealing with infeasibility	273
Reversible activities	277
14.3 Other piecewise-linear functions	279
14.4 Guidelines for piecewise-linear optimization	282
Rules for piecewise-linear expressions	282
Suggestions for piecewise-linear models	282
<b>Chapter 15. Integer Linear Programs</b>	<b>291</b>
15.1 Integer variables	292
15.2 Zero-one variables and logical conditions	293
Fixed costs	294
Zero-or-minimum restrictions	298
Cardinality restrictions	299
15.3 Practical considerations in integer programming	302

<b>Appendix A. AMPL Reference Manual</b>	<b>307</b>
A.1 Lexical rules	307
A.2 Set members	308
A.3 Indexing expressions and subscripts	308
A.4 Expressions	309
A.4.1 Built-in functions	311
A.4.2 Piecewise-linear terms	312
A.5 Declarations of model entities	313
A.6 Set declarations	314
A.6.1 Cardinality function	315
A.6.2 Ordered sets	315
A.6.3 Intervals and other infinite sets	316
A.7 Parameter declarations	317
A.7.1 Check statements	318
A.7.2 Infinity	318
A.8 Variable declarations	318
A.9 Constraint declarations	319
A.10 Objective declarations	320
A.11 Suffix notation for auxiliary values	321
A.12 Standard data format	321
A.12.1 Set data	321
A.12.2 Parameter data	324
A.13 Command language	327
A.13.1 Printing commands: <code>display</code> , <code>print</code> and <code>printf</code>	328
A.13.2 Options and environment variables	330
A.13.3 The <code>solve</code> command	331
A.13.4 The <code>solution</code> command	332
A.13.5 The <code>write</code> command	332
A.13.6 Auxiliary files	333
A.13.7 The <code>objective</code> , <code>drop</code> and <code>restore</code> commands	333
A.13.8 The <code>fix</code> and <code>unfix</code> commands	334
A.13.9 The <code>shell</code> command	334
A.13.10 The <code>reset</code> and <code>update</code> commands	334
A.13.11 The <code>let</code> command	335
A.13.12 The <code>quit</code> and <code>end</code> commands	335
A.14 Imported functions	335
A.15 Defined variables	337
A.16 Reserved and predefined words	338
A.17 Synonyms	339
<b>Index</b>	<b>341</b>