
contents

NOTATION

1	FORMULATION OF LINEAR PROGRAMS	1
1.1	Formulation of linear programs	1
1.2	Solving linear programs in two variables	19
2	THE SIMPLEX METHOD	35
2.1	Introduction	35
2.2	Transforming the problem into the standard form	36
2.3	Canonical tableau	40
2.4	Phase I with a full artificial basis	44
2.5	The simplex algorithm	46
2.6	Outline of the simplex method for a general linear program	52
2.7	The Big- M Method	65
3	THE GEOMETRY OF THE SIMPLEX METHOD	75
3.1	Euclidean vector spaces: Definitions and geometrical concepts	75
3.2	Matrices	84
3.3	Linear independence of a set of vectors and simultaneous linear equations	86
3.4	Basic feasible solutions and extreme points of convex polyhedra	97
3.5	The usefulness of basic feasible solutions in linear programming	102
3.6	The edge path traced by the simplex algorithm	109
3.7	Homogeneous solutions, unboundedness, resolution theorems	114
3.8	The main geometrical argument in the simplex algorithm	124
3.9	The dimension of a convex polyhedron.	124
3.10	Alternate optimum feasible solutions and faces of convex polyhedra	125
3.11	Pivot matrices	128
3.12	Canonical tableaus in matrix notation	130
3.13	Why is an algorithm required to solve linear programs?	131
3.14	Phase I of the simplex method using one artificial variable	136

4	DUALITY IN LINEAR PROGRAMMING	147
4.1	Introduction	147
4.2	An example of a dual problem	147
4.3	Dual variables are the prices of the items	149
4.4	How to write the dual of a general linear program	150
4.5	Duality theory for linear programming	155
4.6	Other interpretations and applications of duality	169
5	REVISED SIMPLEX METHOD	183
5.1	Introduction	183
5.2	Revised simplex algorithm with the explicit form of inverse when Phase II can begin directly	183
5.3	Revised simplex method using Phase I and Phase II	187
5.4	Revised simplex method using the product form of the inverse	192
5.5	Advantages of the product form implementation over the explicit form implementation	197
6	THE DUAL SIMPLEX METHOD	201
6.1	Introduction	201
6.2	Dual simplex algorithm when a dual feasible basis is known initially	201
6.3	Disadvantages and advantages of the dual simplex algorithm	208
6.4	Dual simplex method when a dual feasible basis is not known at the start	208
6.5	Comparison of the primal and the dual simplex methods	215
7	PARAMETRIC LINEAR PROGRAMS	219
7.1	Introduction	219
7.2	Analysis of the parametric cost problem given an optimum basis for some λ	221
7.3	To find an optimum basis for the parametric cost problem	224
7.4	Summary of the results on the parametric cost problem	224
7.5	Analysis of the parametric right-hand side problem given an optimum basis for some λ	225
7.6	To find an initial optimum basis for the parametric right-hand side problem, given a feasible basis for some λ	226
7.7	To find a feasible basis for the parametric right-hand side problem	226
7.8	Summary of the results on the parametric right-hand side problem	227
7.9	Convex and concave function on the real line	227
8	SENSITIVITY ANALYSIS	243
8.1	Introduction	243
8.2	Introducing a new activity	244
8.3	Introducing an additional inequality constraint	245
8.4	Introducing an additional equality constraint	248
8.5	Cost ranging of a nonbasic cost coefficient	249
8.6	Cost ranging of a basic cost coefficient	250

8.7	Right hand side ranging	251
8.8	Changes in the input-output coefficients in a nonbasic column vector	252
8.9	Change in a basic input-output coefficient	253
8.10	Practical applications of sensitivity analysis	253
9	DEGENERACY IN LINEAR PROGRAMMING	259
9.1	Geometry of degeneracy	259
9.2	Resolution of cycling under degeneracy	263
9.3	Summary of the simplex algorithm using the lexico minimum ratio rule	267
9.4	Do computer codes use the lexico minimum ratio rule?	268
10	BOUNDED VARIABLE LINEAR PROGRAMS	271
10.1	Introduction	271
10.2	Bounded variable problems	271
10.3	Simplex method using working bases	274
11	PRIMAL ALGORITHM FOR THE TRANSPORTATION PROBLEM	289
11.1	The balanced transportation problem: Theory	289
11.2	Revised primal simplex algorithm for the balanced transportation problem	299
11.3	The unimodularity property	307
11.4	Labeling methods in the primal transportation algorithm	309
11.5	Sensitivity analysis in the transportation problem	321
11.6	Transportation problems with inequality constraints	325
11.7	Bounded variable transportation problems	329
12	NETWORK ALGORITHMS	341
12.1	Introduction: Networks	341
12.2	Notation	342
12.3	Single commodity maximum flow problems	348
12.4	The primal-dual approaches for the assignment and transportation problems	360
12.5	Single commodity minimum cost flow problems	373
12.6	Shortest route problems	382
12.7	Minimum spanning tree problems	390
12.8	Multicommodity flow problems	391
12.9	Other network algorithms	392
13	FORMULATION OF INTEGER AND COMBINATORIAL PROGRAMMING PROBLEMS	397
13.1	Introduction	397
13.2	Formulation examples	398

14	CUTTING PLANE METHODS FOR INTEGER PROGRAMMING	419
14.1	Introduction	419
14.2	Fractional cutting plane method for pure integer programs	426
14.3	Cutting plane methods for mixed integer programs	432
14.4	Other cutting plane methods	433
14.5	How efficient are cutting plane methods?	434
15	THE BRANCH AND BOUND APPROACH	437
15.1	Introduction	437
15.2	The lower bounding strategy	438
15.3	The branching strategy	440
15.4	The search strategy	441
15.5	Requirements for efficiency	445
15.6	The 0-1 knapsack problem	446
15.7	The traveling salesman problem	449
15.8	The general mixed integer linear program	456
15.9	The set representation problem	458
15.10	0-1 problems	467
15.11	Advantages and limitations	469
15.12	Ranking methods	471
15.13	Notes on the branch and bound approach	478
16	COMPLEMENTARITY PROBLEMS	481
16.1	Introduction	481
16.2	Geometric interpretations	482
16.3	Applications in linear programming	485
16.4	Quadratic programming	486
16.5	Two person games	493
16.6	Complementary pivot algorithm	495
16.7	Conditions under which the algorithm works	503
16.8	Remarks	508
17	NUMERICALLY STABLE FORMS OF THE SIMPLEX METHOD	521
17.1	Review	521
17.2	The <i>LU</i> decomposition	522
17.3	The Cholesky factorization	528
17.4	Reinversions	533
17.5	Use in computer codes	534
18	COMPUTATIONAL EFFICIENCY	535
18.1	How efficient is the simplex algorithm?	535

APPENDIX 1	
THE DECOMPOSITION PRINCIPLE OF LINEAR PROGRAMMING	541
APPENDIX 2	
BENDER'S DECOMPOSITION FOR MIXED INTEGER PROGRAMS	553
APPENDIX 3	
CRAMER'S RULE	557
SELECTED REFERENCES IN LINEAR PROGRAMMING	559
INDEX	561