

# Contents

Preface	ix
Drawing	xi
Exchange and capture in the planar restricted parabolic 3-body problem <i>Martha Alvarez-Ramírez and Joaquín Delgado, Josep Maria Cors</i>	1
1    Historical review of escapes and capture	2
2    Final evolutions in the restricted 3-body problem	5
2.1    Restricted parabolic problem	6
2.2    Hyperbolic restricted problem	7
2.3    Circular restricted problem	7
3    Scope and structure of the paper	8
4    The restricted parabolic 3-body problem in pulsating coordinates	8
5    Gradient-like property of the global flow	12
6    The asymptotic system	16
7    Structure of parabolic escape orbits	18
8    Criteria for elliptic-parabolic motion	20
9    Numerical results	23
Fitting Invariant Curves on Billiard Tables and the Birkhoff-Herman Theorem <i>Edoh Y. Amiran</i>	29
1    Integrability	29
1.1    Physical Integrability	29
1.2    Geometric Integrability	30
2    Definitions	30
3    Convexity	32
4    Folds	32
5    A differentiable limit curve with constant homotopy	33
Construction of Periodic Orbits in Hill's Problem for $C \gtrsim 3^{\frac{4}{3}}$ <i>Edward Belbruno</i>	37
1    Introduction	37
2    Hill's Problem	39
3    Construction of Periodic Orbits by Homotopic Continuation	45
Appendix	60

Are there perverse choreographies?	63
<i>Alain Chenciner</i>	
1 The circulant mass matrix	63
2 Adapted decompositions	65
3 The equations of perversity	67
4 Choreographies with less than 6 bodies	68
5 Choreographies	72
6 Polygonal relative equilibria	73
7 Two questions	75
Blow up of total collision in the tetrahedral non-rotating four body problem	77
<i>Joaquín Delgado, Claudio Vidal</i>	
1 Introduction	78
2 Statement of the problem	78
3 Blow up of total collision	80
4 Central configurations	83
4.1 Linear stability of central configurations	86
The planar (rhomboidal) configuration	87
The spatial (tetrahedral) configuration	88
5 Linearization of the flow at the critical points	89
5.1 Linearization at the critical points $P^\pm$ associated to the planar configuration $p$ .	91
5.2 Linearization at the critical points $E_{1,2}^\pm$ associated to the tetrahedral configurations $e_{1,2}$	91
6 Regularization of single binary collisions	92
Symbolic Dynamics for Transition Tori-II	95
<i>Marian Gidea, Clark Robinson</i>	
1 Introduction	95
2 The Conley index and correctly aligned windows	97
3 Preliminary results	100
4 Proof of the Main Theorem	102
A Survey on Bifurcations of Invariant Tori	109
<i>Heinz Hanßmann</i>	
1 Introduction	109
2 Bifurcations of equilibria	110
2.1 Bifurcations at zero eigenvalues	111
2.2 The Hamiltonian Hopf bifurcation	112
3 Bifurcations of periodic orbits	113
3.1 Bifurcations inherited from equilibria	113
3.2 The Hamiltonian flip bifurcation	114
4 Bifurcations of Floquet-tori	114
4.1 Bifurcations of co-dimension one	116
4.2 Bifurcations of higher co-dimension	117
5 Reducibility	119

Perturbing the Lagrange solution to the general three body problem	123
<i>E. Piña, L. Jiménez-Lara</i>	
1 Introduction	123
2 Hamilton equations	127
3 The lagrange case of the three-body problem	129
4 Dimensionless variables	132
5 Perturbing the Lagrange solution	133
Horseshoe periodic orbits in the restricted three body problem	137
<i>J. Llibre, Mercè Ollé</i>	
1 Introduction	137
2 The restricted three-body problem	138
3 Horseshoe periodic orbits and the invariant manifolds of Lyapunov periodic orbits emanating from $L_3$	139
Instability of Periodic Orbits in the Restricted Three Body Problem	153
<i>Daniel Offin, Wojciech Skoczylas</i>	
1 Introduction	153
2 Planar R3BP, mass normalized to one	155
3 A variational principle for hyperbolicity	158
4 Fixed energy variational problem	161
Szyzgies and the Integral Manifolds of the Spatial $N$ -Body Problem	169
<i>Christopher McCord</i>	
1 Introduction	169
2 The Manifolds	170
3 The Vector Field $\alpha$	173
3.1 Away from Collinear	174
3.2 At Collinear	175
4 Limiting Behavior	175
5 Coordinates near Syzygies	177
6 Homology of the Integral Manifolds	178
7 The Three-Body Problem	179
Dynamics and bifurcation near the transition from stability to complex instability	185
<i>M. Ollé, J. R. Pacha and J. Villanueva</i>	
1 Introduction	185
2 Formulation of the problem and methodology	186
3 Normal form process	187
3.1 The Jordan structure of the monodromy matrix	187
3.2 The quadratic part of the Hamiltonian in the adapted coordinates	188
4 Normal form at higher order	190
5 The resonant normal form	192
6 Unfolding and stability of the bifurcated 2-dimensional tori	193
Invariant Manifolds of Spatial Restricted Three-Body Problems: the Lunar Case	199

*Jesús Palacián, Patricia Yanguas*

1	Introduction	200
1.1	Canonical Variables for the Problem	200
1.2	Aim and Scope of the Paper	203
2	The Normal Form Setting	204
2.1	Making Formal Integrals with Lie Transformations	204
2.2	Change of Co-ordinates	206
3	Passage to a 2DOF System	206
3.1	The Normalisation of Delaunay	206
3.2	Analysis of the Resulting System in $S^2 \times S^2$	209
4	Passage to another 2DOF System	212
4.1	The Elimination of the Node	212
4.2	Analysis of the Resulting System in $\mathbb{R}^6 / (S^1 \times S^1)$	214
5	Passage to a 1DOF: the Integrable Approximation	216
5.1	The Second Normalisation	216
5.2	The Resulting System: Quasi-Periodic Orbits and 2D Tori	217
5.3	New Families of Symmetric Periodic Orbits	221

## Path Integral Quantization of the Sphere 225

*Walter Reartes*

1	Preliminaries	225
1.1	The 1-Step Propagator	225
1.2	The Propagator	229
2	An Idea of the General Case	229
2.1	The 1-Step Propagator	229
2.2	The Propagator	231
3	Case $M = S^n$	232
4	Propagators for $\alpha$ -densities	235

## Non-holonomic systems with symmetry allowing a conformally symplectic reduction 239

*Pedro de M. Rios and Jair Koiller*

1	Introduction	239
2	The contact non-holonomic system	240
3	Almost-poisson brackets via moving frames	241
4	Contact almost-Poisson structure	243
5	The compressed system	244
6	The conformally symplectic structure on the compressed system	246
7	Non-Jacobi for the constrained almost-Poisson	247
8	Non-Jacobi for the constrained almost-Poisson, bis	248
9	The compressed almost-Poisson structure is not conformally symplectic in general	250
10	Conclusions	251