

# Contents

Preface .....	x
M. Symbolic Manipulation Codes – Introduction to MAPLE/MATHEMATICA.....	1
M1. MAPLE – first sessions .....	1
M2. Repetitions, conditionals, expansions and integrations .....	6
M3. Arrays and matrices .....	9
M4. Solving equations.....	11
M5. Hints .....	13
M6. MATHEMATICA – first sessions .....	15
M7. Tables and matrices.....	19
M8. Solving equations.....	21
M9. Hints .....	22
D. Direct Methods .....	25
D1. The functional and its stationarity .....	26
D2. Hamilton’s principle .....	28
D3. Minimum of potential energy .....	31
D4. The Bubnov–Galerkin method .....	32
D5. Beam on elastic foundation .....	35
D6. The Rayleigh–Ritz method .....	36
D7. Master program.....	38
D8. Applications .....	40
D9. Considerations of accuracy .....	41
D10. Mathematical considerations .....	43
D11. Operators .....	44
D12. Energy convergence and minimum theorem .....	46
D13. More on trial functions .....	48
D14. Methods of weighted residual .....	49
D15. The iterative Kantorovich method .....	50

D16. Heat transfer in a plate	52
D17. Constraints	54
D18. Computer-generated Euler–Lagrange equations	56
D19. Two degrees of freedom	58
D20. Continuous systems	59
D21. Automatic derivation of governing equations	61
D22. Nature of extremum	63
D23. Transversality conditions	64
D24. Generalizations	66
D25. Gauss' principle	67
D26. Minimum pressure drag	68
D27. Approximating boundary conditions	71
D28. Use of logical expressions	74
D29. Concluding remarks	76
<b>F. Finite Element Method</b>	<b>77</b>
F1. Element stiffness matrix	77
F2. Energy analysis	82
F3. Truss element	85
F4. Physical meaning of matrices	90
F5. Eigenvalues of stiffness matrix	92
F6. Reference systems	95
F7. Generalizations	98
F8. Assembling	99
F9. Assembling via equilibrium equations	101
F10. Assembling via connectivity	105
F11. Applications to a truss	108
F12. Inhomogeneous beam	114
F13. Applications to a beam	116
F14. Automatic generation of an assembly matrix	117
F15. Optimization	121
F16. Imposition of constraints	124
F17. Free vibrations	128
F18. Plate element	132
F19. Programming for plate element	134
F20. Applications	137
F21. Two-dimensional elasticity, 1	140
F22. Two-dimensional elasticity, 2	145
F23. Heat conduction and related problems	148
F24. The Bubnov–Galerkin formulation	150
F25. Heat transfer in a fin	152
F26. Numerical example	154
F27. Time-history analysis	158

F28. Numerical example . . . . .	160
F29. Natural reference systems . . . . .	163
F30. Serendipity coordinates . . . . .	168
F31. Concept of isoparametric elements . . . . .	170
F32. Three-node isoparametric bar element . . . . .	173
F33. Two-dimensional isoparametric element . . . . .	175
F34. Programming for an isoparametric element . . . . .	177
F35. Special elements . . . . .	179
F36. Constraints and Lagrange multipliers . . . . .	182
F37. Constraints and penalty parameters . . . . .	184
F38. Accuracy, convergence and related subjects . . . . .	186
F39. More on element deficiency . . . . .	188
F40. Symbolic database . . . . .	189
F41. Concluding remarks . . . . .	192
S. Finite Difference Methods . . . . .	193
S1. FD-operators and their accuracy . . . . .	193
S2. More about accuracy . . . . .	195
S3. Sample problem . . . . .	197
S4. Runge-Kutta methods, 1 . . . . .	199
S5. Runge-Kutta methods, 2 . . . . .	201
S6. Transient vibrations . . . . .	203
S7. Multi-step schemes . . . . .	205
S8. Automatic generation of multi-step schemes . . . . .	207
S9. Start-up for multi-step schemes . . . . .	209
S10. Iterative predictor-corrector methods . . . . .	211
S11. Non-linear vibrations . . . . .	212
S12. Newmark's method . . . . .	215
S13. Finite difference equations . . . . .	217
S14. Stability . . . . .	219
S15. Solving non-linear equations . . . . .	221
S16. Boundary-value problems . . . . .	224
S17. Discretization of PDEs . . . . .	225
S18. Explicit scheme . . . . .	226
S19. Implicit scheme . . . . .	228
S20. Stability . . . . .	229
S21. Consistency . . . . .	230
S22. Other FD-schemes . . . . .	231
S23. Treatment of boundary conditions . . . . .	232
S24. Heat conduction in a slab (explicit scheme) . . . . .	234
S25. Heat conduction in a slab (implicit scheme) . . . . .	235
S26. Heat conduction in a solidifying alloy, 1 . . . . .	237
S27. Heat conduction in a solidifying alloy, 2 . . . . .	239

S28. Treatment of irregular boundaries . . . . .	241
S29. Torsion of a shaft, 1 . . . . .	241
S30. Torsion of a shaft, 2 . . . . .	244
S31. Concluding remarks . . . . .	246
W. Workshop . . . . .	247
W1. Analysis of plates, 1 . . . . .	247
W2. Analysis of plates, 2 . . . . .	250
W3. Analysis of a shock-absorber, 1 . . . . .	252
W4. Analysis of a shock-absorber, 2 . . . . .	254
W5. Flow through a duct . . . . .	256
W6. Free vibrations by the Rayleigh–Ritz method . . . . .	258
W7. Applications, 1 . . . . .	259
W8. Applications, 2 . . . . .	260
W9. Free vibrations by the Bubnov–Galerkin method . . . . .	262
W10. Non-linear free vibrations . . . . .	264
W11. Applications of Gauss’ principle . . . . .	266
W12. Heat transfer to fluid . . . . .	268
Problems . . . . .	271
Appendix A: Manipulations with Matrices . . . . .	275
Appendix B: Section-file Correspondence . . . . .	277
Further reading . . . . .	279
Index . . . . .	281