# Contents

*Preface* ix

**1 Algebraic and geometrical methods** 1
- 1.1 Natural numbers 1
- 1.2 Units and dimensional analysis 4
- 1.3 Functional notation 8
- 1.4 Quadratic and higher-order equations 8
- 1.5 Dependent and independent variables 10
- 1.6 Graphical methods 12
- 1.7 Some geometrical methods 14
  - 1.7.1 Similar triangles 14
  - 1.7.2 Triangular graph paper 14
  - 1.7.3 Three-dimensional geometry 15
  - 1.7.4 Circle, ellipse, parabola and hyperbola 17
  - 1.7.5 Plane polar coordinates 21
- 1.8 Factorials and gamma functions 22
- 1.9 Probability 23
- 1.10 Complex numbers 24

**2 Differential calculus** 29
- 2.1 Significance and notation 29
- 2.2 The calculus limit 30
- 2.3 Differentiation of simple functions 34
- 2.4 The use of differentials; implicit differentiation 36
- 2.5 Logarithms and exponentials 38
- 2.6 The chain rule and differentiation by substitution 39
- 2.7 Turning points: maxima, minima and points of inflection 41
- 2.8 Maxima and minima subject to constraint; Lagrange's method of undetermined multipliers 45

2.9 Series 50
2.9.1 Geometric series 51
2.9.2 Power series and Taylor's theorem 51
2.9.3 Maclaurin's theorem 54
2.9.4 Inversion of a series 58
2.9.5 Empirical curve fitting by power series 60
2.10 The evaluation of limits by L'Hôpital's rule 61
2.11 The principles of Newtonian mechanics 63

**3 Differential calculus in three or more dimensions; partial differentiation 73**
3.1 Significance and notation 73
3.2 An alternative approach to calculus 77
3.3 The total differential 78
3.4 General expression for a total differential 80
3.5 Exact differentials 82
3.6 Relations between partial derivatives 84
3.7 Extensive and intensive variables; Euler's theorem 86
3.8 Taylor's theorem in partial derivatives 88
3.9 Vectors 89
3.9.1 Scalar and vector products 92
3.9.2 Scalar and vector fields 95

**4 Integration 98**
4.1 Significance and notation 98
4.2 Standard methods of integration 101
4.2.1 Simple functions 101
4.2.2 Reciprocal functions 101
4.2.3 Integration by parts 102
4.2.4 Integration by substitution 104
4.2.5 Expansion of algebraic functions 104
4.2.6 Integration by partial fractions 105
4.3 Standard forms of integral and numerical methods 106
4.4 Multiple integration 107
4.5 Differentiation of integrals; Leibnitz's theorem 108
4.6 The Euler–Maclaurin Theorem 109

**5 Applications of integration** **115**
5.1 Plane area 115
5.2 Plane elements of area 117
5.3 Elements of volume; polar coordinates in three dimensions 120
5.4 Line integrals 122
5.5 Curve length by integration 123
5.6 Applications of multiple integration 124
5.6.1 The pressure of a perfect gas 124
5.6.2 Interactions in a real fluid 127
5.7 The calculus of variations 129
5.8 Generalized dynamics 132

**6 Differential equations** **138**
6.1 Significance and notation 138
6.2 Equations of first order, first degree 140
6.2.1 Separable variables 140
6.2.2 First-order homogeneous equations 142
6.2.3 Exact equations 144
6.2.4 Linear equations of first order 145
6.3 Linear differential equations 147
6.3.1 Homogeneous linear equations with constant coefficients 148
6.3.2 General linear equation with constant coefficients 156
6.3.3 Linear equations of second order 159
6.4 Integral transforms 160
6.4.1 Laplace transforms 160
6.4.2 Fourier transforms 163

**7 Experimental error and the method of least squares** **165**
7.1 Significance 165
7.2 Root-mean-square error 166
7.3 Distribution of error 168
7.4 The statistical analysis of experimental data 169
7.5 Propagation of error 170
7.6 Small-sample errors 171

7.7 The normal distribution of error 174
7.8 The method of least squares 176
7.8.1 Linear relation between two variables 176
7.8.2 Covariance and correlation coefficient 179
7.8.3 Uncertainty in the slope and intercept of the least-squares straight line 180
7.8.4 Least-squares straight line with both variables subject to error 183
7.8.5 Weighting of observations 183
7.8.6 Multivariable and non-linear least-squares analysis 184

*Appendix* SI units, physical constants and conversion factors; the Greek alphabet and a summary of useful relations 185

*Index* 190