Contents

Chapter 1	Som	e Basic Concepts	
_	1.1	Introduction	1
	1.2	Sets and set language	1
	1.3	One-to-one correspondence of sets	5
	1.4	Infinite sets	9
	1.5	Concept of a whole number	11
	1.6	Geometry	14
		Chapter Test	18
Chapter 2	Basic Ideas of Addition and Subtraction of Whole Numbers		
	2.1	Union of sets	21
	2.2	Some remarks on the language of logic	24
	2.3	Definition of the addition of whole numbers	24
	2.4	Intersection of sets	29
	2.5	Subtraction and set complementation	31
	2.6	Subtraction as related to addition	33
	2.7	Number lines	34
	2.8	The use of frames	36
	2.9	Number activities	39
	2.10	Discovery	42
		Chapter Test	44
		Reading: The addition table: Experiences in	
		practice-discovery	48
Chapter 3	Naming Whole Numbers		
	3.1	Early beginnings	55
	3.2	Concept of a base	55
	3.3	Base five number language	56

	3.4 Numerals in base ten	58		
	3.5 Picturing base ten numerals for children	58		
	3.6 Numerals in base five	61		
	3.7 Changing from one base to another	62		
	Chapter Test	67		
	Reading: Early Mayan mathematics	69		
Chapter 4	Addition and Subtraction Algorithms for Whole Number	'S		
	4.1 Introduction	73		
	4.2 Addition with stick bundles and abacus	76		
	4.3 The addition algorithm in symbols	78		
	4.4 Subtraction with stick bundles and abacus	80		
	4.5 Subtraction algorithms in symbols	82		
	4.6 Addition and subtraction in other bases	86		
	Chapter Test	92		
	Reading: A new algorithm for subtraction?	98		
Chapter 5		um-		
	bers	400		
	5.1 Primary-school beginnings	103		
	5.2 Multiplication in terms of Cartesian products of sets	104		
	5.3 Division as partition and as successive subtraction	108		
	5.4 Division as the inverse of multiplication	110		
	5.5 Properties of multiplication and division	112		
	5.6 Number activities involving multiplication and division	117		
	Chapter Test	119		
	Reading: Zero, the troublemaker	122		
Chapter 6	Multiplication and Division Algorithms for Whole Numbers			
	6.1 Introduction	127		
	6.2 Multiplication algorithms	128		
	6.3 Division algorithms	132		
	6.4 Multiplication and division in other bases	134		
	6.5 The G.C.D. and the L.C.M.	136		
	Chapter Test	141		
	Reading: A simplified presentation for finding	1.4.4		
	the L.C.M. and the G.C.F.	144		
	Summary Test for Chapters 1 through 6	146		
Chapter 7	The Integers			
	7.1 Introduction	149		
	7.2 Addition of integers	152		
	7.3 Subtraction of integers	155		
	7.4 Multiplication of integers	159		
	7.5 Division of integers	161		

			Contents	ix
	7.6 A	formal approach to inequalities for integers	1	.62
		bsolute value	_	.66
		hapter Test	1	67
		eading: Grisly grids		70
Chapter 8	More o	on Geometry		
	8.1 In	troduction	1	175
	8.2 D	efinitions in geometry	1	176
	8.3 Co	ongruence and similarity	1	178
	8.4 Te	essellations	1	182
	8.5 Th	ne geoboard	1	185
	8.6 G	eometry of space	1	189
	8.7 Di	iscovery in geometry	1	91
	Cl	hapter Test	2	201
	R_{i}	eading: Geometric activities for early child-		
		hood education	2	206
Chapter 9	Rations	al and Irrational Numbers		
C. Leipter		hat are rational and irrational numbers?	2	215
		quivalent fractions		218
		omments on our definitions	2	220
		erminology	_	221
		ecimal fractions	2	223
		actions in bases other than ten		231
		egative rational numbers		233
		equalities and absolute value		234
		formal approach to rational numbers	2	238
	9.10 Pe		2	239
		hapter Test	2	241
		eading: The equation method of teaching percentage		245
~ 10				
Chapter 10	_	tations with Rational Numbers	_	
		troduction	_	249
		ddition of rational numbers		249
		abtraction of rational numbers		252
		ultiplication of rational numbers		255
		ivision of rational numbers		260
		omputations with rational numbers written in bases		
		an ten		266
		atio and proportion		267
		roperties of inequalities		270
		he field properties		270
		hapter Test		271
	R	eading: Addition of unlike fractions	2	274

x Contents

Chapter 11	Measurement			
-	11.1 Introduction	279		
	11.2 Length	281		
	11.3 Area	287		
	11.4 Volume	294		
	11.5 Angles	296		
	11.6 Other measurements	297		
	Chapter Test	299		
	Reading: Grids, tiles, and area	302		
Chapter 12	Recapitulation			
	Sets	310		
	Number and number systems	310		
	Structure and properties	310		
	Numeration	311		
	Sentences	311		
	Operations	311		
	Geometry	311		
	Reasoning and proof	311		
	Problem solving	312		
	Measurement	312		
	Summary Test for Chapters 7 through 12	313		
Chapter 13	Binary Operations			
	13.1 Examples and definitions	317		
	13.2 Properties of binary operations	319		
	13.3 Clock arithmetic	320		
	Chapter Test	325		
	Reading: An application of modular number systems	327		
Chapter 14	Functions			
	14.1 The "Guess My Rule" game	329		
	14.2 Function "machines"	330		
	14.3 Definition of a function	333		
	14.4 Functions as mappings	336		
	14.5 Functions as sets of ordered pairs	338		
	14.6 Graphs of functions	340		
	Chapter Test	346		
	Reading: Let's consider the function!	349		
Chapter 15	Transformation Geometry			
	15.1 Moving figures in a plane	355		
	15.2 Translations	356		
	15.3 Rotations	358		
	15.4 Reflections	362		

		Contents	xi
	15.5 Combining movements		366
	15.6 Symmetries of figures in a plane		368
	Chapter Test		370
	Reading: Informal geometry through symmetry		375
Chapter 16	Probability		
	16.1 Introduction		381
	16.2 Experimental probability		381
	16.3 The sample space of an experiment		383
	16.4 Events		385
	16.5 Definition of probability		387
	16.6 Some properties of $P(E)$		390
	16.7 A counting principle		392
	Chapter Test		395
Chapter 17			
	17.1 Introduction		399
	17.2 Organizing statistical data		399
	17.3 Graphical representation of statistical data		402
	17.4 Measures of central tendency		408
	Chapter Test		412
	Reading: Graphs in the primary grades		416
Chapter 18	Number Theory		
	18.1 Introduction		421
	18.2 Tests for divisibility		422
	18.3 Casting out nines		424
	18.4 Some unsolved problems in number theory		426
	18.5 Fibonacci numbers		431
	Reading: Modular arithmetic		438
	List of Symbols		443
	Answers to Summary Tests and Selected Exercises		445
	Index		457