

CONTENTS

1. HISTORICAL INTRODUCTION	
1. Newton	1
2. Maclaurin	3
3. Jacobi	4
4. Meyer and Liouville	7
5. Dirichlet, Dedekind, and Riemann	8
6. Poincaré and Cartan	11
7. Roche, Darwin, and Jeans	12
Bibliographical Notes	13
2. THE VIRIAL EQUATIONS OF THE VARIOUS ORDERS	
8. Introduction	15
9. The moments describing the distribution of density, pressure, and velocity	15
10. The tensor potentials and the potential-energy tensors	17
11. The virial equations of the various orders	20
(a) The equations of the first order	
(b) The equations of the second order	
(c) The equations of the third order	
(d) The equations of the fourth order	
12. The virial equations in a rotating frame of reference	24
(a) The equations of the second order	
(b) The equations of the third order	
13. The variations resulting from small departures from equi- librium. The Lagrangian and the Eulerian changes	28
14. The equations governing small departures from a given initial flow	31
15. The first variations of the various integral properties and the linearized form of the virial equations	32
Bibliographical Notes	37
3. THE POTENTIALS OF HOMOGENEOUS AND HETEROGENEOUS ELLIPSOIDS	
16. Introduction	38

17. Newton's and related theorems on the potential interior to homoeoidal shells	39
18. The potential of a homogeneous ellipsoid at an interior point	43
19. The potential of a homogeneous ellipsoid at an external point. Maclaurin's and Ivory's theorems	45
20. The potentials of heterogeneous ellipsoids	49
21. The index symbols	53
22. The potentials \mathfrak{D}_i , \mathfrak{D}_{ij} , and \mathfrak{D}_{ijk}	55
23. The first variations of the potential-energy tensors	59
(a) The variation $\delta\mathfrak{B}_{ij}$ for homogeneous ellipsoids	
(b) The variation $\delta\mathfrak{B}_{ij;k}$ for homogeneous ellipsoids	
(c) Restrictions on solenoidal displacements	
Bibliographical Notes	62
4. DIRICHLET'S PROBLEM AND DEDEKIND'S THEOREM	
24. Introduction	64
25. The hydrodynamical equations in a moving frame	64
26. The second-order virial equations in a moving frame	66
27. The Riemann-Lebovitz formulation of Dirichlet's problem	67
28. Dedekind's theorem	71
29. The integrals of equation (57)	73
30. The equivalence to the virial equation	74
Bibliographical Notes	75
5. THE MACLAURIN SPHEROIDS	
31. Introduction	77
32. The equilibrium figures	77
33. The second-harmonic modes of oscillation	80
(a) The transverse-shear modes	
(b) The toroidal modes	
(c) The pulsation mode	
(d) The proper solutions for the displacements	
(i) The transverse-shear modes	
(ii) The toroidal modes	
(iii) The pulsation mode	
34. A necessary condition for the occurrence of a point of bifurcation	88
35. The isolation of the point $\Omega^2 = 2B_{11}$ from a consideration of the virial relations	90
36. The stable part of the Maclaurin sequence as a curve of bifurcation in four different ways	91

37. The effect of viscous dissipation on the stability of the Maclaurin spheroid	95
(a) The second-order virial equations allowing for viscous dissipation	
(b) The low Reynolds-number approximation	
(c) The effect of viscous dissipation on the toroidal modes	
Bibliographical Notes	100
6. THE JACOBI AND THE DEDEKIND ELLIPSOIDS	
38. Introduction	101
39. The Jacobi ellipsoids: the equilibrium figures	101
40. The bifurcation of Poincaré's sequence of pear-shaped configurations from the Jacobian sequence	103
(a) A direct determination of the point of bifurcation	
41. The points along the Maclaurin sequence where sequences of pear-shaped configurations bifurcate	110
42. The second- and the third-harmonic oscillations of the Jacobi ellipsoid	112
(a) The second-harmonic oscillations	
(b) The third-harmonic oscillations	
43. The third-harmonic oscillations of the Maclaurin spheroid	117
44. The Dedekind ellipsoids	124
45. The stability of the Dedekind ellipsoids and the point of bifurcation of a sequence of pear-shaped configurations	125
Bibliographical Notes	127
7. THE RIEMANN ELLIPSOIDS	
46. Introduction	129
47. Riemann's theorem	129
48. The equilibrium figures in the case ζ and Ω are parallel: the S -type ellipsoids	132
(a) The adjoint Riemann ellipsoids and Dedekind's theorem	
(b) The stable Maclaurin spheroids as the first members of Riemann sequences	
(c) The bounding self-adjoint sequences	
(d) The irrotational sequence, $\zeta^{(0)} = 0$ and $f = -2$	
(e) Prolate spheroids among the S -type ellipsoids	

49. The stability of the S-type ellipsoids with respect to second-harmonic oscillations	146
(a) The characteristic equation for the even modes of oscillation	
(b) The characteristic equation for the odd modes of oscillation	
(c) The locus of marginal stability	
50. The loci of neutral points belonging to the third harmonics in the domain of occupancy of the S-type ellipsoids	154
51. The Riemann ellipsoids in which the directions of Ω and ζ are not parallel; the ellipsoids of types I, II, and III	156
(a) The domain of occupancy in the $(a_2/a_1, a_3/a_1)$ -plane	
(b) The ellipsoids of type III as branching off from the ellipsoids of type S along a curve of bifurcation	
(c) The Maclaurin spheroids as limiting forms of the Riemann ellipsoids of type I	
(d) The adjoint ellipsoids and Dedekind's theorem	
(e) The disklike ellipsoids on the a_2 -axis	
52. The stability of the ellipsoids of types I, II, and III with respect to second-harmonic oscillations	168
53. A class of finite-amplitude oscillations of the Maclaurin spheroid	172
(a) Configurations with extremum or minimum energy	
(b) Spheroidal oscillations	
(c) Ellipsoidal oscillations	
Bibliographical Notes	184
8. THE ROCHE ELLIPSOIDS	
54. Introduction	189
55. Roche's problem	189
(a) The second-order virial equation appropriate to Roche's problem	
56. The Roche ellipsoids: the equilibrium figures	191
(a) The Jeans spheroids and the tidal problem	
(b) The arrangement of the solutions	
(c) The Roche limit	
57. The stability of the Roche ellipsoids with respect to the second-harmonic oscillations	199
(a) The characteristic equation governing the odd modes of oscillation	
(b) The characteristic equation governing the even modes of oscillation	
(c) The point of onset of dynamical instability along a Roche sequence	

58. The neutral point belonging to the third harmonics	207
59. The effect of viscous dissipation on the stability of the Roche ellipsoid	209
(a) The viscous mode	
(b) The effect of viscous dissipation on the remaining three even modes	
60. The Roche–Riemann ellipsoids	215
(a) The solution for the case $p = 0$	
61. Darwin's problem	218
(a) The second-order virial equation appropriate to Darwin's problem	
62. The Darwin ellipsoids	221
(a) The case $M/M' = 0$	
(b) The case of congruent components	
63. The stability of the congruent Darwin ellipsoids	229
(a) The nature of the synchronous oscillations	
(b) The tidal potential between two slightly misaligned ellipsoids	
(c) Equations governing the fluid motions in the ellipsoids appropriate for treating synchronous oscillations	
(d) The virial equations governing the synchronous oscillations of the Darwin ellipsoids	
(e) The reduction of the second-order virial equation	
(f) The reduction of the first-order virial equations	
(g) The dynamical instability of the congruent Darwin ellipsoids	
(h) The 'Roche limit	
Bibliographical Notes	240
EPILOGUE	241
SELECTED REFERENCES	243
APPENDIX: LIST OF PAPERS	245
SUBJECT INDEX	249
INDEX OF SYMBOLS	253