CONTENTS

1	Introduction to dynamical systems	1
	1.1 Mathematical models	1
	1.2 Sets of mathematical objects	5
	1.3 The space \mathbb{R}^2	12
	1.4 Optimal control problems	14
	1.5 The space \mathbb{R}^n	17
	Exercises	19
	Notes for Chapter 1	19
2	Aspects of set theory	22
	2.1 Unions and intersections	22
	2.2 Equivalence relations and classes	29
	2.3 Congruence modulo n (with $n \in \mathbb{N}$)	35
	Exercises	37
	Notes for Chapter 2	37
3	Mappings	40
	3.1 Introduction	40
	3.2 General mappings	46
	3.3 Special mappings	47
	3.4 Inverse mappings	52
	Exercises	57
	Notes for Chapter 3	58
4	Semigroups and groups	60
	4.1 Binary operations	60
	4.2 Semigroups	61
	4.3 Groups	66
	4.4 Isomorphisms and homomorphisms	76
	4.5 Cosets, normal subgroups, and quotient groups	81
	Exercises	87
	Notes for Chapter 4	89
5	Rings and fields	91
_	5.1 Rings	91
	5.2 The polynomial ring $\mathbb{R}[z]$	96
	5.3 Homomorphisms, ideals, and quotient rings	98

xii CONTENTS

	5.4 Fields	104
	Exercises	105
	Notes for Chapter 5	106
6	Vector spaces and modules	108
	6.1 Vector spaces	108
	6.2 Linear independence and bases	113
	6.3 Modules	119
	6.4 Submodules and module homomorphisms	124
	6.5 Torsion modules and free modules	127
	6.6 State-space module X_{Σ}	129
	Exercises	134
	Notes for Chapter 6	135
7	Metric and normed spaces	138
	7.1 Metric spaces	138
	7.2 Normed spaces	142
	Exercises	146
	Notes for Chapter 7	147
8	Limits, convergence, and boundedness	148
	8.1 Convergence of sequences and series	148
	8.2 Supremum and infimum	153
	8.3 Cauchy sequences and completeness	155
	8.4 Uniform convergence	157
	8.5 Generalizations	161
	8.6 Other topics, including Zorn's lemma and the contraction-	
	mapping theorem	166
	Exercises	167
	Notes for Chapter 8	168
9	Sets, convexity, and topology	170
	9.1 Open and closed sets	170
	9.2 Convex sets	178
	9.3 Compact sets	183
	9.4 Null sets	187
	9.5 Topological spaces	189
	Exercises	195
	Notes for Chapter 9	195
10	Continuity and differentiability	197
	10.1 Continuity	198
	10.2 Differentiation	209

	CONTENTS	xiii
	10.3 Differentiable mappings	216
	10.4 Implicit functions	225
	Exercises	232
	Notes for Chapter 10	233
11	Manifolds and Lie algebras	236
	11.1 Vector fields	236
	11.2 Manifolds	242
	11.3 Lie groups	249
	11.4 The singular-control problem	251
	11.5 Linear algebras	254
	Exercises	258
	Notes for Chapter 11	259
Solutions to the exercises		261
References		292
Index		295