

Table of Contents

1	Introduction	1
2	The Basic Principles of Continuation Methods	7
2.1	Implicitly Defined Curves	7
2.2	The Basic Concepts of PC Methods	13
2.3	The Basic Concepts of PL Methods	15
3	Newton's Method as Corrector	17
3.1	Motivation	17
3.2	The Moore-Penrose Inverse in a Special Case	18
3.3	A Newton's Step for Underdetermined Nonlinear Systems ...	20
3.4	Convergence Properties of Newton's Method	22
4	Solving the Linear Systems	28
4.1	Using a QR Decomposition	29
4.2	Givens Rotations for Obtaining a QR Decomposition	30
4.3	Error Analysis	31
4.4	Scaling of the Dependent Variables	34
4.5	Using LU Decompositions	35
5	Convergence of Euler-Newton-Like Methods	37
5.1	An Approximate Euler-Newton Method	37
5.2	A Convergence Theorem for PC Methods	38
6	Steplength Adaptations for the Predictor	44
6.1	Steplength Adaptation by Asymptotic Expansion	45
6.2	The Steplength Adaptation of Den Heijer & Rheinboldt	50
6.3	Steplength Strategies Involving Variable Order Predictors ...	55
7	Predictor-Corrector Methods Using Updating	61
7.1	Broyden's "Good " Update Formula	61
7.2	Broyden Updates Along a Curve	68

8	Detection of Bifurcation Points Along a Curve	75
8.1	Simple Bifurcation Points	75
8.2	Switching Branches Via Perturbation	84
8.3	Branching Off Via the Bifurcation Equation	87
9	Calculating Special Points of the Solution Curve	91
9.1	Introduction	91
9.2	Calculating Zero Points $f(c(s))=0$	92
9.3	Calculating Extremal Points $\min_s f((c(s)))$	94
10	Large Scale Problems	96
10.1	Introduction	96
10.2	General Large Scale Solvers	97
10.3	Nonlinear Conjugate Gradient Methods as Correctors	101
11	Numerically Implementable Existence Proofs	112
11.1	Preliminary Remarks	112
11.2	An Example of an Implementable Existence Theorem	114
11.3	Several Implementations for Obtaining Brouwer Fixed Points	118
11.4	Global Newton and Global Homotopy Methods	123
11.5	Multiple Solutions	128
11.6	Polynomial Systems	132
11.7	Nonlinear Complementarity	141
11.8	Critical Points and Continuation Methods	145
12	PL Continuation Methods	151
12.1	Introduction	151
12.2	PL Approximations	156
12.3	A PL Algorithm for Tracing $H(u) = 0$	159
12.4	Numerical Implementation of a PL Continuation Algorithm ..	163
12.5	Integer Labeling	168
12.6	Truncation Errors	171
13	PL Homotopy Algorithms	173
13.1	Set-Valued Maps	173
13.2	Merrill's Restart Algorithm	181
13.3	Some Triangulations and their Implementations	186
13.4	The Homotopy Algorithm of Eaves & Saigal	194
13.5	Mixing PL and Newton Steps	196
13.6	Automatic Pivots for the Eaves-Saigal Algorithm	201
14	General PL Algorithms on PL Manifolds	203
14.1	PL Manifolds	203
14.2	Orientation and Index	211

14.3	Lemke's Algorithm for the Linear Complementarity Problem .	214
14.4	Variable Dimension Algorithms	218
14.5	Exploiting Special Structure	229
15	Approximating Implicitly Defined Manifolds	233
15.1	Introduction	233
15.2	Newton's Method and Orthogonal Decompositions Revisited .	235
15.3	The Moving Frame Algorithm	236
15.4	Approximating Manifolds by PL Methods	238
15.5	Approximation Estimates	245
16	Update Methods and their Numerical Stability	252
16.1	Introduction	252
16.2	Updates Using the Sherman-Morrison Formula	253
16.3	QR Factorization	256
16.4	LU Factorization	262
P1	A Simple PC Continuation Method	266
P2	A PL Homotopy Method	273
P3	A Simple Euler-Newton Update Method	288
P4	A Continuation Algorithm for Handling Bifurcation	296
P5	A PL Surface Generator	312
P6	SCOUT — Simplicial Continuation Utilities	326
P6.1	Introduction	326
P6.2	Computational Algorithms	328
P6.3	Interactive Techniques	333
P6.4	Commands	335
P6.5	Example: Periodic Solutions to a Differential Delay Equation	337
	Bibliography	346
	Index and Notation	383