

# Contents

<b>1</b>	<b>Introduction to Population Dynamics</b>	<b>1</b>
1.1	Elements of Classical Genetics . . . . .	1
1.2	General Evolutionary Equations . . . . .	8
1.3	Two-level Populations . . . . .	16
<b>2</b>	<b>Elementary Models</b>	<b>23</b>
2.1	The Free Single Locus Population . . . . .	23
2.2	The Free Two Locus, Two Allele Population . . . . .	28
2.3	The Dynamic Effects of Sex Linkage . . . . .	36
2.4	Selection at a Two Allele, Autosomal Locus . . . . .	51
2.5	Mutation . . . . .	57
2.6	Appendix: Asymptotic Estimates in One-Dimensional Dynamics . . . . .	59
<b>3</b>	<b>Algebra Foundations</b>	<b>65</b>
3.1	Nonassociative Algebras. Idempotents and Nilpotents . . . . .	65
3.2	Nilalgebras . . . . .	71
3.3	Baric Algebras and Spaces . . . . .	73
3.4	Bernstein Algebras . . . . .	86
3.5	Genetic Algebras and Train Algebras . . . . .	107
3.6	Duplication of an Algebra . . . . .	125
3.7	Stochastic Spaces . . . . .	129
3.8	Stochastic Algebras . . . . .	150
3.9	The Kin of an Evolutionary Algebra. Normalization . . . . .	158
<b>4</b>	<b>Stationary Gene Structure</b>	<b>169</b>
4.1	Statement of Problem. Examples . . . . .	169
4.2	Zygote Genotypes. Principle of Gene Inheritance . . . . .	174
4.3	Elementary Gene Structure (E.G.S.) . . . . .	183
4.4	Nonnegativity for an Arbitrary S.G.S. . . . .	190

4.5	Genetic Criterion for E.G.S. . . . . .	197
4.6	Nonelementary S.G.S. . . . . .	202
<b>5</b>	<b>The General Bernstein Problem</b>	<b>211</b>
5.1	The Necessity of Mendel's First Law . . . . .	211
5.2	Theorem on Constant Inheritance . . . . .	214
5.3	The Nonnegative Projection Associated with an Isolated Idempotent . . . . .	216
5.4	A Theorem on Two Non-Splitting Types . . . . .	218
5.5	The Solution of the Bernstein Problem for Exceptional Populations . . . . .	221
5.6	Small Dimensions . . . . .	225
5.7	Estimate of the Number of Constant Subpopulations. Ultranormal Bernstein Populations . . . . .	228
5.8	Appendix. The Proof of Topological Lemma 5.7.2 . . . . .	233
<b>6</b>	<b>Recombination Processes</b>	<b>235</b>
6.1	Linkage Distribution. Chromosome Structures . . . . .	235
6.2	Evolutionary Equations. Reiersöl Algebra . . . . .	244
6.3	Structure of the Set of Equilibrium States. Convergence to Equilibrium . . . . .	248
6.4	Exact Linearization. Evolutionary Spectrum . . . . .	252
6.5	Explicit Evolutionary Formula . . . . .	256
6.6	The Rate of Convergence to Equilibrium . . . . .	265
6.7	Combining Recombination and Mutation . . . . .	268
6.8	Appendix. Recombination Among Sex Chromosomes . . . . .	269
<b>7</b>	<b>Evolution in Genetic Algebras</b>	<b>275</b>
7.1	Reiersöl Algebras are Genetic . . . . .	275
7.2	Idempotents. Convergence of Trajectories . . . . .	277
7.3	Exact Linearization. Evolutionary Spectrum . . . . .	280
7.4	The Explicit Evolutionary Formula . . . . .	286
<b>8</b>	<b>General Quadratic Evolutionary Operators</b>	<b>291</b>
8.1	The Set of Equilibrium States . . . . .	291
8.2	Contracting Quadratic Operators . . . . .	300
8.3	An Example of Irregular Trajectory Behavior . . . . .	303

<b>9</b>	<b>Selection Dynamics</b>	<b>307</b>
9.1	Evolutionary Selection Equations for an Autosomal Multiallele Locus . . . . .	307
9.2	Stability of Equilibrium States. Fundamental Theorem . . . .	310
9.3	Variance Estimates of Disequilibrium . . . . .	316
9.4	Convergence to Equilibrium for Selection in an Autosomal Multiallele Locus . . . . .	321
9.5	Evolutionary Selection Equations in a System of Autosomal Multiallele Loci . . . . .	329
9.6	Convergence to Equilibrium under Additive Selection for a System of Autosomal Multiallele Loci . . . . .	334
9.7	Appendix. Characterization of Additive Selection . . . . .	343
<b>A</b>	<b>Commentaries and References</b>	<b>347</b>
	<b>Index</b>	<b>371</b>