Preface

vii

Introduction	n	1
Chapter 1.	The principle of virtual displacement. Problem of mathematical programming.	
1.1.	Introduction	5
1.2.	Equilibrium of systems under bilateral constraints. Generalized coordinates.	8
1.3.	Equilibrium of systems under bilateral and unilateral constraints. The principle of virtual displacement.	17
Chapter 2.	The detachment principle and optimization methods	
2.1.	Introduction	25
2.2.	The method of multipliers	27
2.3.	Equilibrium of systems under elastic constraints. The method of penalty functions	32
2.4.	Equipotential conditions	38
2.5.	The development of the method of penalty functions	40
Chapter 3.	The energy theorem	
3.1.	The energy theorem	45
3.2.	Consequences of the energy theorem. The principle of maximum work.	49
3.3.	The principle of minimum work	54
3.4.	Linear programming	58
3.5.	Modelling of constraints by force fields and the energy theorem	60
3.6.	The problem of choice of elasticity parameters of constraints in constrained optimization problems	63
Chapter 4.	Models for systems of linear equations and inequalities. Alternative theorems. Models for linear programming problems.	
4.1.	Introduction	68
4.2.	Linear equations and inequalities	68

х					CONTENTS
	1 3	Dharadaal	 £	 _	

4 5		
4.3.	Physical models for systems of linear equations and inequalities	71
4.4.	Alternative theorems	76
4.5.	Models for linear programming problem. The duality theorem	91
4.6.	Two-way estimates of solutions of linear programming problems	95
4.7.	The method of surplus constraints	9 7
4.8.	The method of surplus variables	99
Chapter 5.	Hodograph method for linear programming problems	
5.1.	Introduction	109
5.2.	The hodograph method for linear programming problems	111
5.3.	Solution of the dual problem	123
5.4.	The hodograph method for the canonical form of linear programming problems	124
5.5.	On the starting problem in the hodograph method	138
Chapter 6.	Method of shifting elastic constraints for linear programming problems	
6.1.	Introduction	141
6.2.	The first algorithm	143
6.3.	The second algorithm	147
6.4.	A combined algorithm	154
Chapter 7.	Problem of maximum flow in networks	
7.1.	Formulation of the problem	159
7.2.	A model for the maximum flow problem	161
7.3.	Conditions for equilibrium. Minimum cut. Duality theorem	165
7.4.	A physical model for the maximum flow problem	167
7.5.	Principles of minimum work	169
7.6.	Algorithms for numerical solution of the problem of maximum flow in a network	172

Chapter 8.	Models and methods for solving transportation problem of linear programming	
8.1.	Model for the transportation problem, Conditions for equilibrium. Duality theorem	184
8.2.	Method of surplus variables and an algorithm for numerical solution of the transportation problem	190
8.3.	Algorithm for solving the dual transportation problem	197
Chapter 9.	Methods of decomposition of linear programming problems	
9.1.	Introduction	209
9.2.	First decomposition algorithm	211
9.3.	On application of the detachment principle to the problem of resource distribution	224
9.4.	Second decomposition algorithm and the resource distribution problem	232
Chapter 10.	Gradient methods	
10.1.	Introduction	239
10.2.	Constrained minimization problems	240
10.3.	Linear programming	247
10.4.	Dynamic problems of optimal control	255
Chapter 11.	The method of aggregation of constraints	
11.1.	Introduction	263
11.2.	Nonlinear programming	264
11.3.	Linear programming	276
11.4.	Optimal aggregation of constraints	279
Chapter 12.	Foundations of thermodynamics	
12.1.	Introduction	294
12.2.	Main features	294
12.3.	The first law of thermodynamics	296
12.4.	The second law of thermodynamics	298
12.5.	The principle of minimum work. The Gibbs potential	308

•	principle.	211
12.7.	Living nature and the principles of thermodynamics	314
Chapter 13.	Equilibrium and distribution of resources	
13.1.	Introduction	317
13.2.	Concept of equilibrium in phenomenal models of transfer and distribution of resources	318
13.3.	Equilibrium of systems with additive effects	3 23
Chapter 14.	Models of economic equilibrium	
14.1.	Introduction	333
14.2.	Equilibrium problem for linear model of exchange	334
14.3.	Algorithm for numerical solution of equilibrium problem for linear exchange model	346
14.4.	Equilibrium of linear economic model	3 50
14.5.	Physical model of economic	3 30
	equilibrium problem. The equilibrium theorem	3 57
14.6.	Algorithm for solving the equilibrium problem of linear economic model	3 64
14.7.	On some generalizations of economic equilibrium problems	368
Chapter 15.	Von Neumann's model of economic growth	
15.1.	Introduction	372
15.2.	Technological and economic growth rates	373
15.3.	Method for solving problem of maximizing growth rate	377
15.4.	Duality of problems of economic and technological growth rates	381
15.5.	Problem of rapid action for model of a developing economy	387
15.6.	Physical model and solution of the rapid action problems	390
15.7.	Decomposition of rapid action problem	393

12.6. The Boltzmann principle

15.8.	Problem of exit to path of a balanced growth	395
15.9.	Main line properties and Saint- Venant's principle	399
Chapter 16.	Analytical dynamics	
16.1.	Introduction	402
16.2.	D'Alembert's principle	403
16.3.	Holonomic systems. Lagrange's equations	406
16.4.	Equations of motion in the case of holonomic additional constraints	412
16.5.	Nonholonomic systems	414
16.6.	Hamilton's principle	419
16.7.	The case where force function of field depends on velocities	4 23
16.8.	Hamilton's canonical form of equations of motion	424
16.9.	Elimination of cyclic coordinates	429
16.10.	Parametric form of canonical equations. Extension of phase space	431
Chapter 17.	Dynamics of systems under elastic constraints	
17.1.	Introduction	435
17.2.	The Hamilton principle. Equations of motion	436
17.3.	Discussion	442
17.4.	Methods of shifting deformable constraints	4 51
17.5.	Method of successive approximations	453
Chapter 18.	Dynamical problems of optimal control	
_	Introduction	457
18.2.	Formulation of the problem	458
18.3.	Method of shifting nonholonomic deformable constraints	462
18.4.	Maximum principle	467
18.5.	Methods for numerical solution of optimal control problems	480

xiv