Contents

I PRELIMINARY 1

§1 Introduction 1

1.1 Statement of the Problem 1
1.2 Examples 2
1.3 Outline 3

§2 An Appropriate Concept of Measure 6

2.1 Q-Measure 6
2.2 Lemmata 7

II INTEGRAL CHARACTERIZATIONS OF GLOBAL OPTIMALITY 9

§1 Mean Value Conditions 9

1.1 Mean Value over Level Sets 10
1.2 A Limit-Based Definition 14
1.3 Mean Value Conditions 16

§2 Variance and Higher Moment Conditions 17

2.1 Variance over Level Sets 18
2.2 A Limit-Based Definition 23
2.3 Variance Conditions 24
2.4 Higher Moments 26
2.5 Higher Moment Conditions 29

§3 The Constrained Cases 32

3.1 Rejection Conditions 32
3.2 Reduction Conditions 34
3.3 Linear Equality Constraints 37

§4 Penalty Global Optimality Conditions 45

4.1 Penalty Mean Value 45
4.2 Penalty Mean Value Conditions 49
4.3 Penalty Variance and Higher Moment Conditions 50

§5 Convex Programming 54

5.1 Optimality Conditions for Differentiable Convex Functions 54
5.2 Optimality Lemmas 56
5.3 Optimality Conditions for Convex Minimization 59
5.4 Generalized Gradient 63

§6 Optimality Conditions for Differentiable Functions 67

6.1 General Discussion: the Unconstrained Case 68
6.2 The Inequality Constrained Case in $\mathbb{R}^n$ 70
6.3 Equality and Inequality Constrained Cases in $\mathbb{R}^n$ 72

§7 Integer and Mixed Programming 74

7.1 Integer Minimization Problems 74
7.2 Optimality Conditions 75
7.3 Mixed Minimization Problems 77

§8 Optimality Conditions for a Class of Discontinuous Functions 80

8.1 Robust Sets 80
8.2 The Structure of a Robust Set on the Real Line $\mathbb{R}$ 83
8.3 Robust Continuity 84
8.4 Optimality Conditions 86

III THEORETICAL ALGORITHMS AND TECHNIQUES 88

§1 The Mean Value-Level Set (M-L) Method 88

1.1 Algorithm 89
1.2 Convergence 90
1.3 The Actual Descent Property 91
1.4 The Influence of Errors 93

§2 The Rejection and Reduction Methods 95

2.1 The Rejection Method 95
2.2 The Reduction Method 97
2.3 The Reduction Method for Linear Equality Constrained Cases in $\mathbb{R}^n$ 99

§3 Global SUMT and Discontinuous Penalty Functions 101

3.1 SUMT and the Set of Global Minima 102
3.2 Discontinuous Penalty Functions 106

§4 The Nonsequential Penalty Method 108

4.1 Construction 109
4.2 Convergence 113

§5 The Technique of Adaptive Change of Search Domain 114

5.1 A Simple Model 115
5.2 Convergence 116
5.3 Optimality Conditions of the Simple Model 118
5.4 The General Model 120

§6 Stability of Global Minimization 122

6.1 Continuity of Mean Value 122
6.2 Stability of Global Minima 125

§7 Lower Dimensional Approximation 128

7.1 Approximation of Global Minimum 129
7.2 Estimation of Degree of Approximation 130

IV MONTE CARLO IMPLEMENTATION 132

§1 A Simple Model of Implemention 133

1.1 The Model 133
1.2 Monte Carlo Implementation 135
1.3 The Flow Chart 136

§2 Statistical Analysis of the Simple Model 137

2.1 Estimators of the Search Domains 137
2.2 The Probability of Escape and the Sample Size 139
2.3 Asymtotic Estimation of the Amount of Computation 140

§3 Strategies of Adaptive Change of Search Domains 143

3.1 Strategies 143
3.2 The Change of Domain Theorem 145
3.3 Reduction of the Skew Rate 147

§4 Remarks on Other Models 148

4.1 Rejection and Reduction Models 148
4.2 Integer and Mixed Programming 150
4.3 The Multi-Solution Model 151

§5 Numerical Tests 152

V APPLICATIONS 156

§1 Unconstrained Problems 156

1.1 Automatic Design of Optical Thin Films 156
1.2 Optimal Design of an Equalizer Network 160

§2 Applications of the Rejection Method 161

2.1 Optimal Design of Optical Phase Filters 162
2.2 Optimal Design of an Automatic Transmission Line Attenuation Compensation Network 163

§3 Applications of the Reduction Method 167

3.1 Optimal Design of a Turbine Wheel 168
3.2 Nonlinear Observation and Identification 170

§4 An Application of the Penalty Method 173

4.1 Weight Minimization of a Speed Reducer 173

§5 An Application of Integer and Mixed Programming 175

5.1 Optimal Design of an Optical Thin Film System 175

Bibliography 177