Contents

General Introduction 11

0.1 The reform of mathematics teaching 11
0.2 New and old 12
0.3 The reform of content and teaching method 13
0.4 Mathematics as a tool and as an autonomous science 14
0.5 Mathematics as an art 16
0.6 Mathematics as a whole 16
0.7 Familiarity and surprise 20
0.8 Levels 21
0.9 Cycles in learning mathematics 23
0.10 Individual differences 25
0.11 Discovery and exposition 27
0.12 Motivation 28
0.13 Evaluation 29
0.14 Towards a science of mathematical education 30
0.15 Programmed learning 31
0.16 The growing role of the teacher 32
References 32

Part One New Ways of Teaching: Practice and Theory 35

1 Classroom Treatment of Some Essential Topics 37

1.1 A lesson in logic 38
1.2 The concept of a group 46
1.3 Glide reflections 53
1.4 Greatest common divisor and lowest common multiple 60
1.5 The use of environmental interests in developing a project 67
1.6 A lesson on linear programming 70
1.7 Probability and statistics 75
1.8 Matrices 79
1.9 Calculus 83
References 93

2 The Use of Teaching Aids 94

2.0 Introduction 94
2.1 Drawings and diagrams 95
2.2 Geo-boards 97
2.3 Three-dimensional models 98
2.4 Space frames 99
2.5 Moving models 99
2.6 Films 100
2.7 Algebraic aids 105
2.8 Logical materials 108
2.9 Calculating machines 111
2.10 Textbooks 118
2.11 The mathematics laboratory 119
References 120

3 Treatment of the Axiomatic Method in Class 124

3.1 Intuitive meaning of axioms 125
3.2 Axiomatization: description and definition 129
3.3 Preparatory stage 138
3.4 Axiomatic organization of the course 139
3.5 Some problems in teaching the development of the axiomatic theory 142
3.6 Problems of rigour 143
3.7 Confrontation: the traditional and modern conceptions of the axiomatic method in teaching 145
References 150

4 Psychological and Educational Research Bearing on Mathematics Teaching 151

4.0 The field 151
4.1 Intellectual ability with special reference to mathematics 152
4.2 Studies of the growth of pupils' thinking 154
4.3 Concept formation, productive thinking and problem solving 161
4.4 Learning mathematics 167
4.5 Curricular studies 170
4.6 Summary 171
References 171

Part Two The New Mathematics 179

5 A Selection of Syllabuses 181

5.1 Belgium 183
5.2 Denmark 187
5.3 Federal Republic of Germany 192
5.4 Japan 201
5.5 Sweden 203
5.6 Union of Soviet Socialist Republics 205
5.7 United Kingdom 208
5.8 United States of America 211

6 A Modern Secondary-School Syllabus in Mathematics for the Scientific Stream 217

6.0 Introduction 217
6.1 Documentation 219
6.2 A modern assignment of mathematical subject matter 220
6.3 First year (fifteen and sixteen years old) 228
6.4 Second year (sixteen and seventeen years old) 230
6.5 Third year (seventeen and eighteen years old) 231

Part Three Steps to Reform 233

7 The Training and Re-Training of Mathematics Teachers 235

7.1 Training primary teachers 235
7.2 Training secondary teachers 237
7.3 Further training of teachers 245
7.4 Teachers and parents 249
References 252