

# CONTENTS

<b>NOTATIONAL CONVENTIONS</b>	<b>xiii</b>
<b>GLOSSARY OF KEY SYMBOLS</b>	<b>xiv</b>
<b>PREFACE</b>	<b>xvii</b>
<b>1 INTRODUCTION</b>	<b>1</b>
1.1 Historical Perspective	3
1.2 Sunspot Numbers	13
1.3 A Test Case	16
1.4 Issues in Spectral Estimation	17
1.5 How to Use This Text	21
References	22
<b>2 REVIEW OF LINEAR SYSTEMS AND TRANSFORM THEORY</b>	<b>25</b>
2.1 Introduction	25
2.2 Signal Notation	26
2.3 Continuous Linear Systems	26
2.4 Discrete Linear Systems	28
2.5 Continuous-Time Fourier Transform	31
2.6 Sampling and Windowing Operations	34
2.7 Relating the Continuous and Discrete Transforms	37
2.8 The Issue of Scaling for Power Determination	42
2.9 The Issue of Zero Padding	43
2.10 The Fast Fourier Transform	44
2.11 Resolution and the Time-Bandwidth Product	46
References	49
Problems	50
Appendix 2.A Source of Complex-Valued Signals	51
Appendix 2.B Wavenumber Processing with Linear Spatial Arrays	53
Appendix 2.C Fast Fourier Transform Program	54
<b>3 REVIEW OF MATRIX ALGEBRA</b>	<b>57</b>
3.1 Introduction	57
3.2 Matrix Algebra Basics	57
3.3 Special Vector and Matrix Structures	61
3.4 Matrix Inverses	67
3.5 Least Squares Normal Equations	69
3.6 Solution of Linear Equations	71

3.7 Eigenanalysis and Singular Value Decomposition	73
3.8 The Toeplitz Matrix	80
References	92
Problems	94
Appendix 3.A Program to Solve Hermitian Linear Equations by Cholesky Method	96
Appendix 3.B Program to Find Complex Singular Value Decomposition	98
Appendix 3.C Program of the Levinson Algorithm	104
Appendix 3.D Program to Solve General Toeplitz Linear Equations	105
Appendix 3.E Program to Solve Hermitian Toeplitz Linear Equations	107
Appendix 3.F Program to Solve for Minimum Eigenvalue and Eigenvector of a Hermitian Toeplitz Matrix	109
<b>4 REVIEW OF RANDOM PROCESS THEORY</b>	<b>111</b>
4.1 Introduction	111
4.2 Probability and Random Variables	111
4.3 Random Processes	115
4.4 Ergodicity: From Ensemble Averages to Time Averages	121
4.5 Entropy Concepts	125
References	126
Problems	126
Appendix 4.A Bias and Variance of the Sample Spectrum	127
<b>5 CLASSICAL SPECTRAL ESTIMATION</b>	<b>130</b>
5.1 Introduction	130
5.2 Summary	131
5.3 Windows	136
5.4 Resolution and the Stability-Time-Bandwidth Product	144
5.5 Autocorrelation and Cross Correlation Estimation	146
5.6 Correlogram Method Power Spectral Density (PSD) Estimators	149
5.7 Periodogram PSD Estimators	152
5.8 Combined Periodogram/Correlogram Estimators	158
5.9 Application to Sunspot Numbers	161
5.10 Conclusion	164
References	165
Problems	166
Appendix 5.A Program to Compute Correlation Estimates	167
Appendix 5.B Program to Compute the Correlogram Method PSD Estimate	168
Appendix 5.C Program to Compute the Periodogram Method PSD Estimate	170
<b>6 PARAMETRIC MODELS OF RANDOM PROCESSES</b>	<b>172</b>
6.1 Introduction	172
6.2 Summary	173
6.3 Autoregressive (AR), Moving Average (MA), and Autoregressive-Moving Average (ARMA) Random Process Models	174
6.4 Relationships Among AR, MA, and ARMA Model Parameters	178

- 6.5 Relationship of AR, MA, and ARMA Parameters to the Autocorrelation Sequence 181
- 6.6 Spectral Factorization 184
  - References 185
  - Problems 186
  - Appendix 6.A Program to Compute the ARMA, AR, and MA Power Spectral Density 186

## **7 AUTOREGRESSIVE PROCESS AND SPECTRUM PROPERTIES 189**

- 7.1 Introduction 189
- 7.2 Summary 190
- 7.3 Autoregressive Process Properties 191
- 7.4 Autoregressive Power Spectral Density Properties 198
  - References 203
  - Problems 204

## **8 AUTOREGRESSIVE SPECTRAL ESTIMATION: BLOCK DATA ALGORITHMS 206**

- 8.1 Introduction 206
- 8.2 Summary 207
- 8.3 Correlation Function Estimation Methods 209
- 8.4 Reflection Coefficient Estimation Methods 211
- 8.5 Least Squares Linear Prediction Estimation Methods 216
- 8.6 Estimator Characteristics 224
- 8.7 Model Order Selection 229
- 8.8 Autoregressive Processes with Observation Noise 231
- 8.9 Application to Sunspot Numbers 232
  - References 235
  - Problems 237
  - Appendix 8.A Program of Yule-Walker Algorithm 239
  - Appendix 8.B Program of Burg (Harmonic) Algorithm 240
  - Appendix 8.C Fast Algorithm and Program to Solve Covariance Equations of Linear Prediction 241
  - Appendix 8.D Fast Algorithm and Program to Solve the Modified Covariance Equations of Linear Prediction 251

## **9 AUTOREGRESSIVE SPECTRAL ESTIMATION: SEQUENTIAL DATA ALGORITHMS 261**

- 9.1 Introduction 261
- 9.2 Summary 262
- 9.3 Gradient Adaptive Autoregressive Methods 264
- 9.4 Recursive Least Squares (RLS) Autoregressive Methods 266
- 9.5 Fast Lattice Autoregressive Methods 272
- 9.6 Application to Sunspot Numbers 273
  - References 274
  - Problems 275

Appendix 9.A Program of Shift Register	275
Appendix 9.B Program of LMS Adaptive Algorithm	276
Appendix 9.C Fast RLS Algorithm and Program to Solve Exponentially Windowed Equations of Linear Prediction	276
<b>10 AUTOREGRESSIVE-MOVING AVERAGE SPECTRAL ESTIMATION</b>	<b>285</b>
10.1 Introduction	285
10.2 Summary	286
10.3 Moving Average Parameter Estimation	288
10.4 Separate Autoregressive and Moving Average Parameter Estimation	290
10.5 Simultaneous Autoregressive and Moving Average Parameter Estimation	294
10.6 Sequential Approach to ARMA Estimation	295
10.7 A Special ARMA Process for Sinusoids in White Noise	296
10.8 Application to Sunspot Numbers	297
References	298
Problems	299
Appendix 10.A Program for Estimating the Parameters of a Moving Average Model	300
Appendix 10.B Program for Estimating the Parameters of an Autoregressive-Moving Average Model	301
<b>11 PRONY'S METHOD</b>	<b>303</b>
11.1 Introduction	303
11.2 Summary	304
11.3 Simultaneous Exponential Parameter Estimation	306
11.4 Original Prony Concept	308
11.5 Least Squares Prony Method	310
11.6 Modified Least Squares Prony Method	312
11.7 Prony Spectrum	315
11.8 Accounting for Known Exponential Components	318
11.9 Identification of Exponentials in Noise	319
11.10 Application to Sunspot Numbers	324
References	325
Problems	326
Appendix 11.A Fast Algorithm and Program to Solve Symmetric Covariance Normal Equations	327
Appendix 11.B Program for Factoring a Complex Polynomial	335
Appendix 11.C Program of Prony Method	344
Appendix 11.D Program for Computing the Exponential Parameters	347
Appendix 11.E Program for Prony Energy Spectral Density	349
<b>12 MINIMUM VARIANCE SPECTRAL ESTIMATION</b>	<b>350</b>
12.1 Introduction	350
12.2 Summary	351
12.3 Derivation of the Minimum Variance Spectral Estimator	352

12.4	Relationship of Minimum Variance and Autoregressive Spectral Estimators	354
12.5	Implementation of the Minimum Variance Spectral Estimator	357
12.6	Application to Sunspot Numbers	357
	References	358
	Problems	358
	Appendix 12.A Program of Minimum Variance Spectral Estimator	359
<b>13</b>	<b>EIGENANALYSIS-BASED FREQUENCY ESTIMATION</b>	<b>361</b>
13.1	Introduction	361
13.2	Summary	362
13.3	Eigenanalysis of Autocorrelation Matrix for Sinusoids in White Noise	362
13.4	Eigenanalysis of Data Matrix for Exponentials in Noise	366
13.5	Signal Subspace Frequency Estimators	368
13.6	Noise Subspace Frequency Estimators	371
13.7	Order Selection	374
	References	374
	Problems	376
	Appendix 13.A Program to Compute the Eigenvector Method and MUSIC Method Frequency Estimators	376
<b>14</b>	<b>SUMMARY OF SPECTRAL ESTIMATORS</b>	<b>379</b>
	Synopsis Table	381
	References	384
<b>15</b>	<b>MULTICHANNEL SPECTRAL ESTIMATION</b>	<b>386</b>
15.1	Introduction	386
15.2	Summary	387
15.3	Multichannel Linear Systems Theory	388
15.4	Multichannel Random Process Theory	389
15.5	Multichannel Classical Spectral Estimators	392
15.6	Multichannel ARMA, AR, and MA Processes	394
15.7	Multichannel Yule-Walker Equations	397
15.8	Multichannel Levinson Algorithm	400
15.9	Multichannel Block-Toeplitz Matrix Inverse	402
15.10	Multichannel Autoregressive Spectral Estimation	403
15.11	Autoregressive Order Selection	409
15.12	Experimental Comparison of Multichannel AR PSD Estimators	409
15.13	Multichannel Minimum Variance Spectral Estimation	416
15.14	Two Channel Spectral Analysis of Sunspot Numbers and Air Temperature	417
	References	420
	Problems	421
	Appendix 15.A Program for Multichannel Periodogram	423
	Appendix 15.B Program for Multichannel AR Estimation by Nuttall-Strand or Vieira-Morf Algorithms	424
	Appendix 15.C Program to Compute Multichannel Autoregressive PSD	427
	Appendix 15.D Supporting Subroutines for Subroutines	

<b>16 TWO-DIMENSIONAL SPECTRAL ESTIMATION</b>	<b>432</b>
16.1 Introduction	432
16.2 Summary	443
16.3 Two-Dimensional Linear Systems and Transform Theory	434
16.4 Two-Dimensional Random Process Theory	440
16.5 Classical 2-D Spectral Estimation	442
16.6 Modified Classical 2-D Spectral Estimation	445
16.7 Two-Dimensional Autoregressive Spectral Estimation	445
16.8 Two-Dimensional Maximum Entropy Spectral Estimation	457
16.9 Two-Dimensional Minimum Variance Spectral Estimation	458
References	459
Problems	461
Appendix 16.A Program to Compute the 2-D Periodogram	462
Appendix 16.B Program to Compute the First-Quadrant and Second- Quadrant 2-D Autoregressive Parameters	463
Appendix 16.C Program to Compute the First-Quadrant 2-D Autoregressive Spectrum	465
<b>17 EPILOGUE</b>	<b>467</b>
<b>APPENDIX I: SUNSPOT NUMBERS</b>	<b>469</b>
<b>APPENDIX II: COMPLEX TEST DATA</b>	<b>476</b>
<b>APPENDIX III: TEMPERATURE DATA</b>	<b>478</b>
<b>APPENDIX IV: PROGRAM CONVERSION FOR COMPLEX-TO-REAL CASE</b>	<b>482</b>
<b>APPENDIX V: LIST OF COMPUTER PROGRAMS</b>	<b>483</b>
<b>INDEX</b>	<b>485</b>