## **CONTENTS**

Ы	PREFACE		xiii
1	IN	TRODUCTION	1
2	NO	ONLINEAR OPTIMIZATION PROBLEMS	7
	2.1	Historical survey of nonlinear optimization	7
	2.2	Classical nonlinear optimization problems and optimality conditions	12
	2.3	Convex optimization	18
	2.4	Separation theorems	23
3	OF	PTIMALITY CONDITIONS	27
	3.1	Smooth nonlinear optimization problems	27
	3.2	Necessary optimality conditions	29
	3.3	Sufficient optimality conditions	34
4	GEOMETRIC BACKGROUND OF		
		OPTIMALITY CONDITIONS	37
	4.1	Geometric meaning of optimality conditions	37
	4.2	Classical differential geometric aspects	41
5	DE	DUCTION OF THE CLASSICAL	
		OPTIMALITY CONDITIONS IN	
		NONLINEAR OPTIMIZATION	45
	5.1	First-order necessary conditions under equality constraints	45
	5.2	Second-order conditions under equality constraints	48

	5.3	Necessary and sufficient conditions under inequality constraints	51
	5.4	A second-order sufficient condition	54
6	GE	ODESIC CONVEX FUNCTIONS	61
	6.1	Geodesic convex functions on Riemannian manifolds	63
	6.2	First-order characterization	72
	6.3	Second-order characterization	75
	6.4	Optimality conditions and geodesic convexity	80
	6.5	Geodesic convexity in nonlinear optimization	81
	6.6	Concluding remarks	85
7	ON	THE CONNECTEDNESS OF	
		THE SOLUTION SET TO	
		COMPLEMENTARITY SYSTEMS	87
	7.1	Linear complementarity systems	89
	7.2	The case of LCS with one parameter	94
	7.3	Nonlinear complementarity systems	96
	7.4	Generalized nonlinear complementarity systems	101
	7.5	Variational inequalities	102
	7.6	Image problem	106
	7.7	Concluding remarks	108
8	NC	ONLINEAR COORDINATE	
		REPRESENTATIONS	111
	8.1	Formulation of the problem	112
	8.2	Nonlinear coordinate representations in $\mathbb{R}^n$	114
	8.3	Right inverses and projections	117
	8.4	Inverse of partitioned matrices by right inverses	127
	8.5	Nonlinear coordinate representations in constrained optimization	130
	8.6	Nonlinear coordinate representations of Riemannian metrics	133
	8.7	Convexification by nonlinear coordinate transformations	135
	8.8	Image representations	135
	8.9	Concluding remarks	139
		~	

Contents ix

9	TE	NSORS IN OPTIMIZATION	141
	9.1	Tensors	142
	9.2	Tensors in coordinate representations	145
	9.3	Smooth unconstrained optimization problems	148
	9.4	Improvement of the structure of global optimization problems	153
	9.5	Smooth constrained optimization problems	157
	9.6	Tensor approximations of smooth functions on Riemannian manifolds	160
	9.7	Tensor field complementarity systems	162
	9.8	Concluding remarks	165
10	GE	ODESIC CONVEXITY ON $\mathbb{R}^n_+$	167
	10.1	Geodesic convexity with respect to the affine metric	169
	10.2	Geodesic convexity with respect to other Riemannian metrics	177
	10.3	Geodesic convexity of separable functions	180
11	VARIABLE METRIC METHODS ALONG		
		GEODESICS	185
	11.1	General framework for variable metric methods on Riemannian submanifolds in $\mathbb{R}^n$	186
	11.2	Convergence of variable metric methods along geodesics	190
	11.3	Rate of convergence for variable metric methods along geodesics	196
	11.4	Variable metric methods along geodesics under inequality constraints	200
	11.5	An optimization approach for solving smooth nonlinear complementarity systems $% \left( 1\right) =\left( 1\right) +\left( 1$	204
12	POLYNOMIAL VARIABLE METRIC		
		METHODS FOR LINEAR	
		OPTIMIZATION	207
	12.1	A class of polynomial variable metric algorithms for linear optimization	210
	12.2	Riemannian metric for the affine scaling vector field	223
	12.3	Riemannian metric for the projective scaling vector field	226

13	SPECIAL FUNCTION CLASSES	231		
	13.1 Geodesic quasiconvex functions	231		
	13.2 Geodesic pseudoconvex functions	233		
	13.3 Difference of two geodesic convex functions	235		
	13.4 Convex transformable functions	236		
	13.5 Pseudolinear functions	239		
14	FENCHEL'S UNSOLVED PROBLEM OF			
	LEVEL SETS	253		
	14.1 Fenchel problem of level sets	255		
	14.2 Preference orderings	257		
	14.3 Utility functions of a preference ordering	258		
	14.4 Main results	262		
	14.5 Preliminary Lemmas and Theorems	267		
	14.6 Proof of Theorems 14.4.1, 14.4.1'	269		
	14.7 Proof of Theorem 14.4.2	270		
15	AN IMPROVEMENT OF THE LAGRANGE	<u>r</u>		
	MULTIPLIER RULE FOR SMOOTH			
	OPTIMIZATION PROBLEMS	271		
	15.1 Lagrange multiplier rule for the case of equality constrain	nts 272		
	15.2 Improved Lagrange multiplier rule for the case of equal constraints	ity 274		
	15.3 Improved Lagrange multiplier rule for the case of equal	ity		
	and inequality constraints	280		
	15.4 Some chances of application	283		
$\mathbf{A}$	ON THE CONNECTION BETWEEN			
	MECHANICAL FORCE EQUILIBRIUM	1		
	AND NONLINEAR OPTIMIZATION	285		
	A.1 Statement of the mechanical force equilibrium problem	286		
	A.2 Characterization of the constraints	287		
	A.3 Characterization of a force equilibrium point by the Cou			
	ivron principle	289		
	A.4 Characterization of a force equilibrium point by the print ple of virtual work	ici- 290		

	A.5	Relation between the principle of virtual work and the Court- ivron principle	293
	A.6	Possible velocities in the case of time-dependent constraints	293
	A.7	Relation between the principle of virtual work and the Courtiv- ron principle in the case of time-dependent constraints	301
	A.8		303
$\mathbf{B}$	то	POLOGY	305
	B.1	Topological spaces	305
	<b>B.2</b>	Metric spaces	312
	B.3	Continuous mappings and homeomorphisms	315
	<b>B.4</b>	Subspaces and product spaces	318
	B.5	Compactness	321
	B.6	Connectedness	325
$\mathbf{C}$	RII	EMANNIAN GEOMETRY	329
	C.1	From the history of differential geometry	329
	C.2	Riemannian manifolds	331
	C.3	Geodesic convex functions	336
	C.4	Riemannian manifolds in Euclidean spaces	338
RE	FEI	RENCES	341
ΑŢ	J <b>TH</b>	OR INDEX	363
$\mathbf{SU}$	BJE	CT INDEX	367
N(	)TA'	ΓΙΟΝS	373